THE ARCHITECTURE OF ARTIFICIAL GRAVITY: MATHEMATICAL MUSINGS ON DESIGNING FOR LIFE AND MOTION IN A CENTRIPETALLY ACCELERATED ENVIRONMENT

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THE ARCHITECTURE OF ARTIFICIAL GRAVITY: MATHEMATICAL MUSINGS ON DESIGNING FOR LIFE AND MOTION IN A CENTRIPETALLY ACCELERATED ENVIRONMENT

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Abstract

Habitats support life; life demands motion. This paper uses mathematical derivations and computer simulations to examine environmental design for life and motion in artificial gravity. Although artificial gravity appears increasingly natural as the radius of rotation approaches infinity, it remains significantly unnatural at a kilometer radius. Environmental design may help the inhabitants to adapt by specifically responding to the unearthliness of the gravity. I propose that appropriate architectural forms should be derived not only from static geometric constraints, but also from the apparent dynamic behavior of hanging, falling, and moving objects, particularly with regard to concepts of verticality, horizontality, and modularity. This study reveals involute and catenary curves. If properly incorporated into the architecture, these curves may provide visual and tactile cues to aid the inhabitants in comprehending and adapting to their distorted gravity environment.

Nomenclature

Boldface indicates vector quantities; italics indicate scalar quantities; dots above indicate derivatives with respect to time:

X,Y,Z	Global inertial coordinates.
X,Y,Z	Local environment coordinates.
i,j,k	Basis vectors.
Ω	Angular velocity of x,y,z relative to X,Y,Z .
ω	Angular velocity of object relative to X,Y,Z .
λ	Angular velocity of object relative to <i>x,y,z</i> .
$\mathbf{R},\dot{\mathbf{R}},\ddot{\mathbf{R}}$	Position, velocity, acceleration relative to <i>X,Y,Z</i> .
$\mathbf{r},\dot{\mathbf{r}}$, $\ddot{\mathbf{r}}$	Position, velocity, acceleration relative to <i>x</i> , <i>y</i> , <i>z</i> .
R,V,A	Magnitudes of $\mathbf{R}, \dot{\mathbf{R}}$, $\ddot{\mathbf{R}}$.
r,v,a	Magnitudes of $\mathbf{r}, \dot{\mathbf{r}}, \ddot{\mathbf{r}}$.
H	Angular momentum.
I	Moment of inertia.
M	Moment.
g,g'	Magnitudes of natural and artificial gravity.
t	Elapsed time.
l,h	Arc distance and height relative to observer.
r_f, r_h	Floor radius, height radius $(r_h = r_f - h)$.
r_c	Radius to midpoint of chord.

Arbitrary constant of integration.

Linear distance.

 θ_1, θ_2

 α

q

C

Acceleration by any force other than gravity produces a sensation of weight. Gravity acting alone produces a sensation of weightlessness. Earth weight results

I. Background

Position angles in equations (5) and (6).

Velocity angle in equation (6).

Ouantity defined in equation (14).

not from the downward pull of gravity, but from the equal and opposite upward push of the ground.

Artificial gravity requires continuous acceleration by some agent other than "natural" gravity. Centripetal acceleration via rotation is the usual choice because it is self-sustaining and independent of the translational motion of the rotating system. But when people or objects move within a rotating reference, they may undergo acceleration in inertial space that is neither intended nor expected. These extra accelerations distort the intended gravitational environment.

For an object in motion relative to a rotating environment, the total acceleration in inertial space is:

$$\ddot{\mathbf{R}} = -\Omega^2 \mathbf{r} + 2\Omega \times \dot{\mathbf{r}} + \ddot{\mathbf{r}} \tag{1}$$

This assumes that the angular velocity is constant, that the environment's center of rotation is not accelerated (except perhaps by gravity), and that tidal effects are negligible.

The first term on the right-hand side of equation (1)

 $[-\Omega^2\,r\,]$ represents the centripetal acceleration associated with the angular velocity of the environment. The second term $[2\,\Omega\times\dot{r}\,]$ represents the Coriolis acceleration associated with the relative velocity of the object. The third term $[\ddot{r}\,]$ represents the relative acceleration of the object. For circular motion at constant speed around the circumference of a rotating cylinder, the formula may be written.

$$\mathbf{r} \perp \mathbf{\Omega} , \dot{\mathbf{r}} \perp \mathbf{\Omega} , \dot{\mathbf{r}} \perp \mathbf{r}$$

$$\lambda = \frac{\mathbf{r} \times \dot{\mathbf{r}}}{r^2}$$

$$\dot{\mathbf{r}} = \lambda \times \mathbf{r}$$

$$\ddot{\mathbf{R}} = -\Omega^2 \mathbf{r} + 2\Omega \times \dot{\mathbf{r}} - \lambda^2 \mathbf{r}$$

$$= -(\Omega + \lambda)^2 \mathbf{r}$$
(2a)

In the design of rotating artificial gravity environments, only the first term in equation (2a) $[-\Omega^2 \mathbf{r}]$ represents "design gravity". The others represent gravitational distortions that result from the motion of people and objects within the environment. The goal is to design the environment such that the first term alone yields some preferred acceleration while simultaneously minimizing the others. Equation (2a) suggests three strategies:

1. Restrict the direction of local motion to be parallel to the axis of rotation. In this case the second and third terms of equation (2a) vanish. Unfortunately, eccentric motion parallel to the axis tends to desta-

bilize the rotation — causing the axis to wobble and invalidating the initial assumption of constant angular velocity.

- Minimize the speed of local motion. This seems to imply some type of behavior modification, whether through individual experience and aversion to motion sickness, deliberate training, or some type of mechanical restraint.
- 3. Minimize the angular velocity of the environment. For a given centripetal acceleration, minimizing the angular velocity requires maximizing the radius, as dictated by the first term of equation (2a). The net effect is to minimize the other terms.

Equations (1) and (2) describe linear accelerations. Of equal importance are angular accelerations and changes in momentum. The moment required to produce a change in angular momentum is:

$$\mathbf{M} = \dot{\mathbf{H}}_{XYZ}$$
$$= \dot{\mathbf{H}}_{XYZ} + \mathbf{\Omega} \times \mathbf{H}$$
(3)

After a lifetime of learning to coordinate one's motions on earth, the first term of equation (3) $[\dot{\mathbf{H}}_{XYZ}]$ is expected, but the second $[\boldsymbol{\Omega} \times \mathbf{H}]$ is not. As with the Coriolis acceleration, the unexpected term is a cross product of the angular velocity of the environment.

Assume that x,y,z axes are fixed in the rotating environment, with x aligned with the environment's rotation axis and z aligned along a radius. For an object that is axially symmetric about z, the moment required to rotate about z can be computed as follows:

$$\mathbf{\omega}_{X} = \mathbf{\Omega}$$

$$\mathbf{H} = I_{X} \mathbf{\omega}_{X} + I_{Z} \mathbf{\omega}_{Z}$$

$$\dot{\mathbf{H}}_{XYZ} = I_{Z} \dot{\mathbf{\omega}}_{Z}$$

$$\mathbf{M} = I_{Z} \dot{\mathbf{\omega}}_{Z} + \mathbf{\omega}_{X} \times (I_{X} \mathbf{\omega}_{X} + I_{Z} \mathbf{\omega}_{Z})$$

$$= I_{Z} \dot{\mathbf{\omega}}_{Z} + I_{Z} (\mathbf{\omega}_{X} \times \mathbf{\omega}_{Z})$$
(4)

The rotations about the x and z axes are said to be "cross-coupled".

Experiments with human subjects indicate that illusions of body and visual field angular motions are approximately proportional to the cross product of the angular velocities of the environment and the head. 1,2 To the extent that the head qualifies as "axially symmetric", this is described by the second term of equation (4) $[I_Z\left(\mathbf{w}_X\times\mathbf{w}_Z\right)]$. Of course, this doesn't begin to describe the complex physiology of the human vestibular system, 3 but it does seem approximately to describe perception. It suggests three strategies for minimizing these disturbances:

- 1. Restrict the orientation of local rotation to be parallel to the rotation of the environment. In this case the second term of equation (4) vanishes.
- 2. Minimize the angular velocity of local rotation.
- 3. Minimize the angular velocity of the environment.

Much research has been done over the past thirty years to determine the comfort zone for artificial gravity; various graphs of the zone recur in the literature. ^{4,5} In light of equations (1) through (4), the design of rotating space stations has generally sought to minimize the angular velocity by maximizing the radius (within economic or technological bounds). Authors have implied, and often stated outright, that "earth-normal" gravity environments are obtainable, provided that the radius of rotation is sufficiently large.

Artists' renditions of rotating space stations and space colonies often depict earth architecture transplanted into enormous toroids, cylinders, and spheres. Except for the unusual landscape, nothing in the architecture bespeaks anything abnormal in the gravity environment. But these static, still-life illustrations fail to capture the essential dynamic nature of artificial gravity. For example: the images often contain no clue as to the direction of rotation.

Deviations from "earth-normalcy" become apparent when one considers the behavior of hanging, falling, and otherwise moving objects as observed by a person rotating with the environment. These deviations are not immediately apparent in the equations above. Along with geometric constraints and the artificiality of the climate, they call for a re-examination of basic architectural concepts in the design of artificial gravity environments.

In order to develop humane designs, the architect must understand the inhabitant's perspective. It is difficult to develop a "feel" for artificial gravity by looking at abstract formulas. One purpose of this paper is to develop more tangible representations. I propose that such representations, incorporated into appropriate architectural forms, may help the inhabitants to orient themselves and adapt to the environment.

II. Directionality

It is convenient at this point to adopt a system for describing the direction of motion within an artificial gravity environment in simple everyday terms:

Up: Radially toward the center.
Down: Radially away from the center.
East: Tangentially with the rotation.
West: Tangentially against the rotation.
North: Ninety degrees left of east.
South: Ninety degrees right of east.

With regard to "north" and "south", inside (concave) and outside (convex) observers may agree on the definitions, but they will disagree on the directions of "left" and "right". It does not help to define the "north pole" as the point at which the rotation appears to be counterclockwise — inside and outside observers will disagree on that. My purpose in adopting these definitions is to preserve their lexicology but to interpret them from the perspective of the inhabitants for whom the environment is designed.

It will also be useful to establish a coordinate system for describing the positions of objects relative to an observer, in a manner similar to common practice on earth. I will use coordinates (l,h), where l is arc length measured at the floor, and h is height measured radially up from the floor (a sort of "reverse polar notation").

III. Verticality, Free-Fall, and the Involute Curve

Hanging, Dropping, and Throwing Balls

As iron filings reveal a magnetic field, so free-falling objects reveal a gravitational field. The unearthliness of artificial gravity is apparent in the deviation of free-falling objects from "earth-normal" behavior. Figure 1 provides a play-by-play comparison of holding and dropping a ball under conditions of natural and artificial gravity.

Natural Gravity	Artificial Gravity
The ball's weight is perceived through resistance to gravity.	The ball's weight is perceived through resistance to inertia.
Holding the ball prevents it from accelerating.	Holding the ball causes it to accelerate centripetally.
Releasing the ball allows it to accelerate.	Releasing the ball allows it to stop accelerating.
The ball falls radially toward the center of the earth, parallel to the apparent gravitational force.	The ball falls tangentially away from the center of rotation, perpendicular to the apparent gravitational force.
The ball accelerates toward the floor.	The floor accelerates toward the ball.

Figure 1: Comparison of Natural and Artificial Gravity

On earth, a plumb bob at rest hangs vertically, and the cord aligns with the gravitational force. (Deflection caused by the earth's rotation is generally insignificant.) If the cord is cut, the weight falls along the line of force revealed by the cord. Verticality can be defined by either the orientation of the hanging cord or the trajectory of the falling weight.

In an artificial gravity environment, a plumb bob at rest hangs radially from the center of rotation, and the cord aligns with the centripetal force. If the cord is cut, the force disappears. The weight does not fall radially as one might expect, but tangentially, perpendicular to the apparent gravitational force. Thus, if verticality in an artificial gravity environment is defined by the orientation of the hanging cord, then the trajectory of the falling weight is not vertical. In fact, to an observer within the environment, the trajectory does not even appear straight.

From an inertial reference, the environment appears as a rotating spool, and the trajectory as a thread pulling tangentially away from it. But from the rotating reference, the environment appears motionless, and the trajectory appears as an involute curve. Figure 2 shows the inertial view. The two silhouettes indicate the initial and final positions of a rotating observer. Figure 3 shows the trajectory as seen by the rotating observer. The dotted lines indicate the ball's inertial trajectory at each instant. If the observer could see the inertial trajectory — and not just the succession of instantaneous end-points — it would appear as a thread unwinding from a spool.

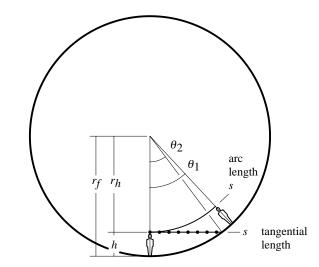


Figure 2: Inertial View of Dropped Ball.

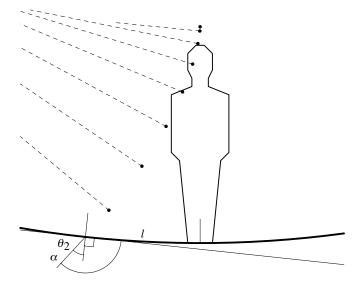


Figure 3: Rotating View of Dropped Ball.

If a ball is held above the floor, and a plumb bob is used to project its initial position onto the floor, then when the ball is dropped it will swerve and strike the floor somewhat west of its initial position. As figure 2 shows, the deflection is completely determined by the geometry: the radius of the floor and the initial height of the ball. It is independent of the rotation rate and acceleration. The initial radial position of the ball, and the tangential distance that it travels before striking the floor, are:

$$r_h = r_f - h$$
$$s = \sqrt{r_f^2 - r_h^2}$$

If the ball was centripetally restrained, its trajectory would be an arc subtending an angle (in radians) of:

$$\theta_1 = \frac{s}{r_h}$$

This is the angle subtended by the centripetally-re-

strained observer. But because the ball is not restrained, its trajectory is a straight line subtending an angle of:

$$\theta_2 = \arctan\left(\frac{s}{r_h}\right)$$

The ball's deflection, measured along the floor, is then:

$$I = r_f \left(\theta_2 - \theta_1\right)$$

$$= r_f \left(\arctan\left(\frac{s}{r_h}\right) - \frac{s}{r_h}\right)$$
(5)

where positive is east and negative is west. The deflection is always to the west, because:

$$\forall x > 0 : \arctan(x) < x$$

The velocity, speed, and direction angle of the falling ball relative to the rotating floor at the point of impact are:

$$\dot{\mathbf{r}} = \Omega \, r_h \left(\cos \left(-\theta_2 \right) \mathbf{i} + \sin \left(-\theta_2 \right) \mathbf{j} \right) - \Omega \, r_f \, \mathbf{i}$$

$$= -\Omega \, \frac{s}{r_f} \left(s \, \mathbf{i} + r_h \, \mathbf{j} \right) \tag{6a}$$

$$V = \Omega S \tag{6b}$$

$$\alpha = -\frac{\pi}{2} - \theta_2$$

$$= -\frac{\pi}{2} - \arctan\left(\frac{s}{r_h}\right)$$
(6c)

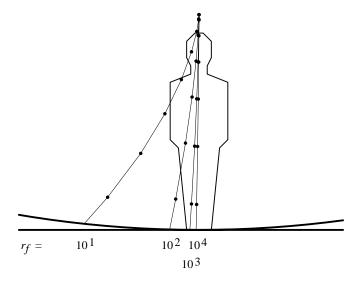
The speed v is directly proportional to the tangential distance of the fall. The direction angle α deviates from vertical by an angle equal in magnitude to the angle subtended by the tangential trajectory.

The elapsed time from the point of release to the point of impact is:

$$t = \frac{s}{\Omega r_h} \tag{7}$$

Figure 4 shows the relationship between floor radius and trajectory deflection for a ball dropped from an initial height of 2 meters. With a floor radius of 1000 meters, the deflection in a 2-meter drop is still more than 8 centimeters.

The behavior of the ball becomes more bizarre if it is forcefully thrown — especially if it is thrown up and against the rotation. Figures 5 and 6 show inertial and rotating views of a ball thrown in artificial gravity. For comparison, figure 7 shows the same ball thrown with the same relative velocity on Earth. Calculation of trajectory, relative velocity at point of impact, and elapsed time is straightforward but tedious: 1) Add the tangential velocity of the environment to the relative velocity of the throw to obtain the total inertial velocity of the ball; 2) Find the intersection of the linear trajectory with the circular floor; 3) Divide the length of the trajectory by the velocity of the ball to obtain the elapsed time; 4) Multiply the angular velocity of the environment by the elapsed time to obtain the rotation of the observer. Because the ball may be given any velocity relative to the environment, the apparent trajectory is not completely



Measurements relative to rotating observer:
Initial Height (m): 2
Initial Velocity (m/s): 0

Floor Radius (m)	Deflection at Floor (m)
10	-1.065
100	-0.272
1000	-0.085

-0.027

10000

Figure 4: Dropping a Ball in Artificial Gravity; Relationship of Floor Radius to Trajectory Deflection

determined by the geometry — it depends as well on the angular velocity of the environment and the relative velocity of the throw. Figure 8 shows a comparison of trajectories for a ball thrown in a particular fashion in a variety of one-g environments.

The involute curve of a falling ball provides a tangible visual image for artificial gravity. Static images like the figures presented here give some indication of its nature, but it is best understood from a dynamic simulation.*

The involute curve represents the natural unforced motion of objects in the artificial gravity environment. It applies not only to falling balls, but also to people, and to their limbs and vestibular organs, as they sit, stand, and walk. Since it is significantly different than the natural motion of objects on earth, it follows that people will

^{*} Figures 2 through 8 are still frames from an interactive graphic simulation that I developed on Apollo computers. In the simulation, the rotation of the space station and the relative trajectory of the ball can be observed in real time. A control panel allows specification of radius, angular velocity, and centripetal acceleration of the space station, and initial height and velocity of the ball

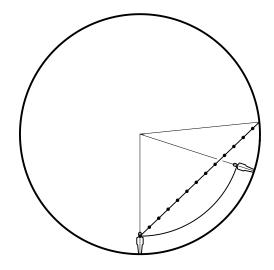


Figure 5: Inertial View of Thrown Ball.

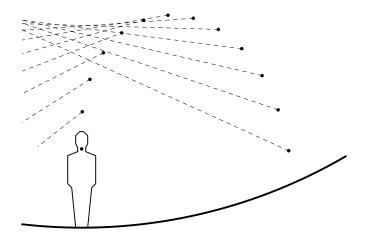


Figure 6: Rotating View of Thrown Ball.

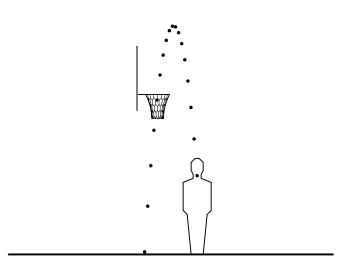
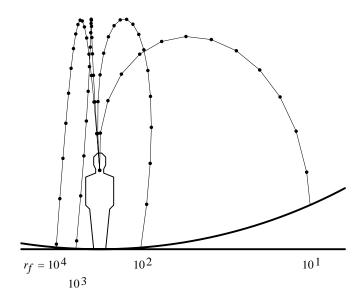


Figure 7: Earth-Normal View of Thrown Ball.



Measurements relative to rotating observer: Initial Height (m): 1.5

Initial Height (m):

Initial Velocity (m/s):

7.5

Initial Inclination from East:

Acceleration (m/s²):

9.8

Floor Radius (m)	Deflection at Floor (m)
10	4.130
100	0.781
1000	-0.454
10000	-0.832
∞	-1.004

Figure 8: Throwing a Ball in Artificial Gravity; Comparison of Trajectories for One-g Environments

move unnaturally, or will need to make an unnatural effort to achieve natural-looking motion. In either case, they will be aware that something is amiss.

Moving through artificial gravity can be likened to traversing a stair: Ascending is different than descending. Neither is a problem for average healthy people provided they see the stair before stepping onto it. Seeing the stair allows them to make appropriate mental and motor adjustments to avoid missteps. Traversing a stair in the dark is much more difficult. It requires probing with toes and groping for handholds before committing to a step. Overestimating the number of steps can be as bad as underestimating.

In the same way, moving through artificial gravity requires appropriate mental and motor adjustments to avoid missteps. Moving west is different than moving east. This is implied algebraically by equations (1) and (2), and is clearly illustrated in figures 3, 4, 6, and 8. As people move through the environment, they will need to coordinate according to their direction of travel. But the gravitational distortions associated with Coriolis accelerations and cross-coupled rotations do not occur until after the movement has begun. They are like darkened stairs that must be tripped over before they are perceived.

Incorporating the involute curve in the architecture would provide visual and tactile cues to allow the inhabitants to adjust their movements before tripping. The curve indicates both the magnitude and direction of the gravitational distortion. Because the shape of the curve for a dropped object is independent of rotation rate or gravity level, it could be built in to the architecture just as the radius itself is built in — perhaps in the shape of wall panels, door and window frames, or other accents.

Limit as Radius Approaches Infinity

A comparison of figures 7 and 8 seems to indicate that artificial gravity becomes increasingly natural as the radius of rotation approaches infinity. With regard to the relative trajectory of thrown objects, this can be shown mathematically.

Celestial mechanics dictates that a ball thrown on Earth will follow an elliptical orbit (albeit perturbed by non-ideal conditions), with one focus at the center of the earth. At the human scale, it is common practice to simplify calculations by assuming that the distance to the center of the earth is infinite. Mathematically, it can be said that the eccentricity of the ellipse approaches one as the radius of the planet approaches infinity. In this sense, the trajectory approaches a parabola as the radius approaches infinity. The parabolic trajectory can be described parametrically as:

$$x(t) = \dot{x}_0 t + x_0 (8a)$$

$$y(t) = -\frac{1}{2}gt^2 + \dot{y}_0t + y_0 \tag{8b}$$

where x_0 , y_0 , \dot{x}_0 , \dot{y}_0 are the initial position and velocity of the ball relative to the observer, g is the gravitational acceleration (9.8 m/s²), and t is the elapsed time

What about artificial gravity? At small radii, the convoluted trajectory looks nothing like an ellipse or parabola.

The coordinate systems used for this analysis are shown in figure 9. The initial position of the ball in the rotating rectangular coordinate system is:

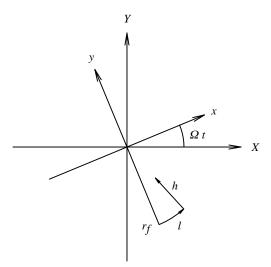


Figure 9: Coordinate Systems Used to Analyze the Relative Trajectory of a Ball in Artificial Gravity

$$x_0 = \left(r_f - h_0\right) \sin\left(\frac{l_0}{r_f}\right) \tag{9a}$$

$$y_0 = -\left(r_f - h_0\right) \cos\left(\frac{l_0}{r_f}\right) \tag{9b}$$

Its initial position and velocity in the inertial coordinate system are:

$$X_0 = x_0$$

$$Y_0 = y_0$$

$$\dot{X}_0 = \dot{x}_0 - \Omega y_0$$

$$\dot{Y}_0 = \dot{y}_0 + \Omega x_0$$

Its positions in the inertial and rotating coordinate systems at time t are:

$$X(t) = \dot{X}_0 t + X_0$$

$$Y(t) = \dot{Y}_0 t + Y_0$$

$$X(t) = X(t)\cos(-\Omega t) - Y(t)\sin(-\Omega t)$$

$$Y(t) = X(t)\sin(-\Omega t) + Y(t)\cos(-\Omega t)$$
(10a)

If the centripetal acceleration at the floor is g', the angular velocity is:

$$\Omega = \left(\frac{g'}{r_f}\right)^{1/2}$$

The functions $\sin(\theta)$ and $\cos(\theta)$ can be written in Maclaurin's series form as:

$$\sin(\theta) = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} \dots$$
$$\cos(\theta) = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} \dots$$

Substituting into equations (9) and (10), canceling terms, and taking the limit as floor radius approaches infinity reveals:

$$\lim_{r_f \to \infty} x_0 = l_0 \tag{11a}$$

$$\lim_{r_f \to \infty} y_0 = -\left(r_f - h_0\right) \tag{11b}$$

$$\lim_{r_f \to \infty} x(t) = \dot{x}_0 t + l_0 \tag{12a}$$

$$\lim_{r_f \to \infty} y(t) = -\frac{1}{2} g' t^2 + \dot{y}_0 t - (r_f - h_0)$$
 (12b)

^{*} A conic section is an ellipse, parabola, or hyperbola depending on whether its eccentricity is less than, equal to, or greater than one. The eccentricity is defined by the ratio of distances from any point on the conic to a fixed point (the focus) and a fixed line (the directrix).

Equations (8) and (12) are analogous. In artificial gravity, as in natural gravity, the trajectory of a free-falling ball approaches a parabola as the radius approaches infinity.

Experiments indicate that human subjects adapt better to artificial gravity environments when the radius is large and the angular velocity is small. (This is shown in graphs of the comfort zone referred to earlier.) This is usually explained in terms of head-to-foot gravity gradients, Coriolis forces, and cross-coupled rotations, as in equations (1) though (4). Equations (8) and (12) provide another interpretation: the unforced relative motion of free-falling objects is more nearly earth-like when the radius of rotation is large.

IV. Linear Motion and the Catenary Curve

Linearity and Modularity

Centripetal acceleration is proportional to radius. Assuming that uniform gravity is a desirable design feature, this implies that the radius should be constant—that the station should have a circular cross section.

Nevertheless, a review of artificial gravity design concepts reveals some uncertainty: Must the cross section be a true circle, or would a many-sided polygon suffice? In 1951, Von Braun proposed a twenty-sided station.⁶ In 1962, NASA Langley Research Center and North American Aviation studied a six-sided station.⁷ In 1975, a rectilinear modular construction system was suggested for the interior of the "Stanford Torus".⁸

For modular design, there are several advantages to approximating a circle with an n-sided polygon. Compared to curved modules, straight modules are:

- easier to pre-fabricate and furnish;
- easier to package and launch;
- more interchangeable, especially between decks at different radii.

A polygon approaches a circle as the number of sides approaches infinity. The problem is to find the minimum number of sides, or the maximum ratio of chord length to radius, to maintain a comfortable gravity environment. This calls for an analysis of linear motion in a rotating environment.

Linearity and Apparent Slope

For linear motion at constant speed relative to a rotating environment, the perceived gravity varies in both magnitude and direction. The total acceleration is the sum of the centripetal and Coriolis accelerations — the first two terms of equation (1).* The centripetal acceleration is always directed toward the center of rotation — it depends only on position, and is independent of speed. The Coriolis acceleration is always perpendicular to the relative motion — it depends only on speed, and is independent of position.

Figure 10 shows the centripetal, Coriolis, and total acceleration of an observer moving west-to-east at constant speed on a flat floor in a rotating environment. "Up" is defined by the direction of the total acceleration. If r_C is the radius to the midpoint of the chord, s is the position on the chord from its midpoint, v is the constant speed, and \mathbf{i} , \mathbf{j} are basis vectors parallel and perpendicular to the chord, then the total acceleration is:

$$\ddot{\mathbf{R}} = -\Omega^2 s \mathbf{i} + \left(\Omega^2 r_C + 2\Omega v\right) \mathbf{j} \tag{13}$$

Figure 11 shows how this observer might perceive his environment. The floor he walks on appears straight, yet its inclination seems to vary as he moves along its length — as if rocking over the top of a hill. The arc length of the hill is equal to the linear length of the floor. The vertical lines under the hill represent the magnitude of the apparent gravity. In the observer's coordinate system, the apparent slope of the linear path is:

$$\frac{dy}{dx} = -\frac{\Omega^2 s}{\Omega^2 r_C + 2\Omega v}$$
$$= -\frac{s}{r_C + \frac{2v}{\Omega}}$$

The shape of the hill is defined by the relationship between slope and arc length (path position):

$$-\left(r_C + \frac{2v}{\Omega}\right)\frac{dy}{dx} = s$$

$$= \int_0^{\overline{X}} \left(1 + \left(\frac{dy}{dx}\right)^2\right)^{1/2} dx$$

Thus, it is an inverted catenary curve of the form:

$$q = r_C + \frac{2v}{\Omega}$$

$$\frac{y}{q} = -\cosh\left(\frac{x}{q}\right) + c \tag{14}$$

where c is an arbitrary constant of integration.

Referring back to figure 10 and equation (13), the magnitude of the acceleration is:

$$A(s) = \left(\left(\Omega^2 s \right)^2 + \left(\Omega^2 r_C + 2 \Omega v \right)^2 \right)^{1/2}$$

In the coordinate system of figure 11, this becomes:

$$A(\bar{x}) = \left(\left(\Omega^2 \int_0^{\bar{x}} \left(1 + \left(\frac{dy}{dx} \right)^2 \right)^{1/2} dx \right)^2 + \left(\Omega^2 r_c + 2 \Omega v \right)^2 \right)^{1/2}$$

$$A(x) = \Omega^2 q \cosh\left(\frac{x}{q} \right)$$
(15)

In figure 11, this acceleration is represented by vertical lines with top endpoint at y = 0 and bottom endpoint at:

$$y = -\frac{A}{\Omega^2}$$

^{*} If the relative speed is not constant, then the third term of equation (1) comes into play. But its contribution to the total perceived gravitation is exactly the same as for linear acceleration on earth.

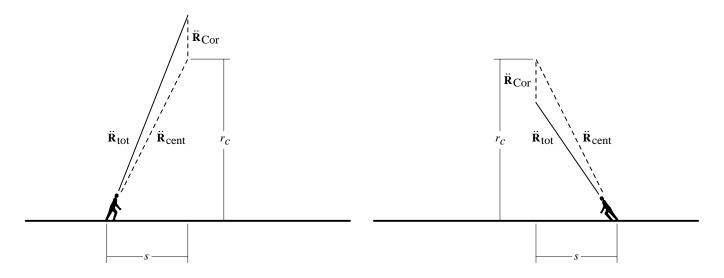


Figure 10: Acceleration of an Observer Walking West-to-East on a Flat Floor in a Rotating Environment

Figure 12: Acceleration of an Observer Walking East-to-West on a Flat Floor in a Rotating Environment

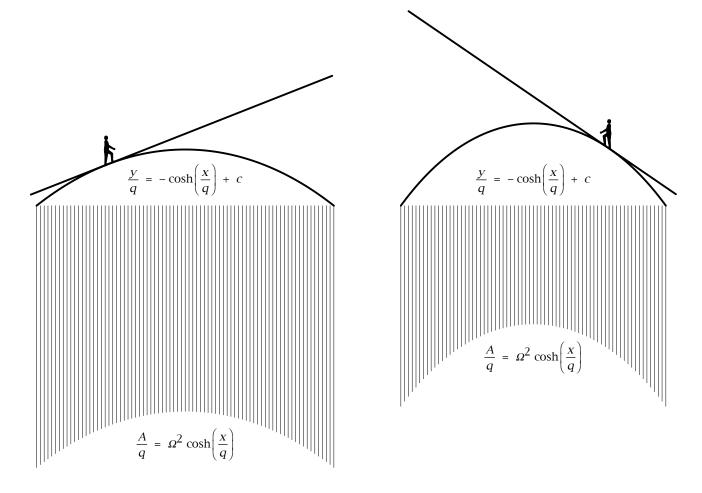


Figure 11: Slope of Floor and Strength of Gravity Perceived by the Observer in Figure 10

Figure 13: Slope of Floor and Strength of Gravity Perceived by the Observer in Figure 12

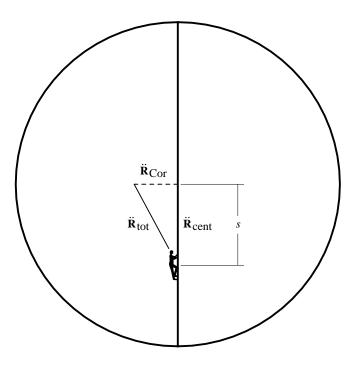


Figure 14: Acceleration of an Observer Climbing a Ladder in a Rotating Environment

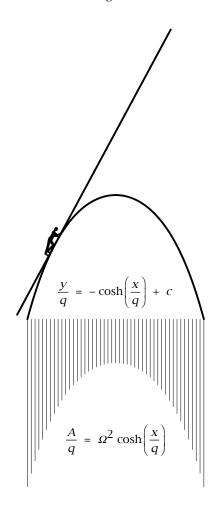


Figure 15: Slope of Ladder and Strength of Gravity Perceived by the Observer in Figure 14

$$\frac{y}{q} = -\cosh\left(\frac{x}{q}\right) \tag{16}$$

Thus, the curve that represents the magnitude of the apparent gravity is a simple translation of the curve that represents the shape of the apparent hill.

Figures 12 and 13 show the situation for an observer walking east-to-west at the same relative speed on the same floor. Notice that the apparent hill is steeper than before. For west-to-east motion, the Coriolis acceleration was added to the normal component of the centripetal acceleration, increasing the effective radius and reducing the apparent slope. For east-to-west motion, the Coriolis acceleration is reversed, reducing the effective radius and increasing the apparent slope.

Figures 14 and 15 show an observer ascending a vertical (radial) ladder. In this case, $r_{\rm C}$ is zero. Such a ladder should be oriented so that it faces east-west (so that its plane is normal to the Coriolis acceleration vector). An observer would find it most comfortable to ascend on the west (as shown) and descend on the east. Therefore, the ladder should be accessible from both sides

Figures 11, 13, and 15 suggest a scheme for studying the comfort parameters for linear elements in an artificial gravity environment. The use of straight cylindrical modules may be particularly advantageous in reconfigurable tethered systems.

V. Island Architecture in the High Frontier

The literature on artificial gravity and space colonization is much concerned with creating an "earthnormal" environment. This is apparent, for example, in the "Island" concept described by O'Neill. Given our lack of experience with artificial gravity, earth-normal environmental design is considered to be conservative but safe.

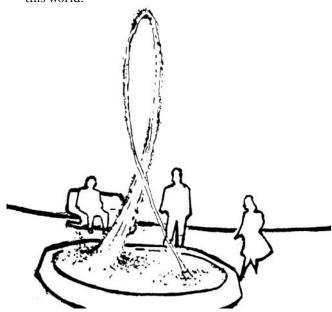
Illustrations of space colonies often depict essentially "normal" earth architecture transplanted to the interiors of enormous toroids, cylinders, and spheres. Inspired by the genre of science fiction and fantasy, the architecture is sometimes vaguely "updated" with assorted curvilinear forms. But these forms seem arbitrary: they are not derived from the ambient environment, nor do they contribute anything to people's adaptation to it.

The study of relative motion indicates that, even though artificial gravity may be earth-force, it will not be earth-normal. Abnormal motion will be noticeable, even if it is not debilitating. The "conservative" approach of transplanting earth architecture into such an environment may actually hinder people's adaptation. It would be presumptuous to propose specific architectural solutions at this stage. The immediate task is to let go of the "earth-normal" hang-up. Once the fallacy of earth-normalcy is abandoned, environmental design for artificial gravity can begin in earnest. The architecture should be as weird as the gravity it's designed for.

At small radii, artificial gravity is as different from natural gravity as weightlessness, and deserves the same attention to detail. Proper environmental design can have a positive impact on crew comfort by reducing the need for off-axis motion and by providing visual cues for orientation to the distorted gravity environment.

At the other extreme, O'Neill's descriptions of space colonies such as "Island Three" (four miles in diameter) invite much speculation:

- In an environment in which air pressure is due primarily to containment rather than weight, would the temperature, pressure, and humidity gradients be conducive to cloud formation? Would it rain? Or fog? Or neither?
- Considering the deflection of free-falling objects in artificial gravity, would a gentle sprinkle become a driving easterly gale by the time it reached the ground?
- What fate would befall a human-powered flyer who strayed too far from the zero-gravity axis?
- The possibilities for fountain design are truly out of this world!



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Revised 1992 (figures 4 and 8).