Artificial Gravity in Theory and Practice

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Rotationally induced artificial gravity is often contested on two fronts: we don't know its physiological effects; and, people can't adapt to it within practical constructible limits of radius and rotation rate. Detractors – including some in the aerospace professional community – sometimes express strong opinions that are not consistent with published research, fundamental theory, or practical experience. To confront those objections, this paper examines the efficacy and implementation of artificial gravity from theoretical and practical perspectives. Contemporary physics posits that all interactions in the universe are attributable to four fundamental forces: strong nuclear, weak nuclear, electromagnetic, and gravitational. The chemical, mechanical, and physiological effects of weight and weightlessness are entirely attributable to the electromagnetic force acting between the electron shells of atoms. The presence of a gravitational field is mostly irrelevant. Other countermeasures to weightlessness, such as exercise, diet, and medications, address myriad individual symptoms but not their underlying cause. With artificial gravity, it is not necessary to enumerate all of the nanoscale dependencies and interactions; it is sufficient to know that the mechanical forces associated with weight are maintained through acceleration, down to the nanoscale of atoms. Living comfortably in rotationally induced artificial gravity might involve a period of adaptation – as does living in weightlessness. It is unreasonable to assert that artificial gravity is unacceptable unless it provides immediate universal comfort – especially considering the significant health benefit it promises. Engineering rotating structures is far less complex than engineering humans to thrive indefinitely in weightlessness.

I. Introduction

THIS paper is motivated by discussions I've had over the years regarding the efficacy and habitability of artificial-gravity environments. I have encountered well-educated people, including physicists, engineers, and architects, working in the aerospace field, with strongly held preconceptions that are inconsistent with established physical theory and contradicted by peer-reviewed published research. Perhaps the distrust of artificial gravity stems in part from the way it's often taught in school, with centrifugal and Coriolis forces mysteriously arising from the ether simply because a structure is rotating, described by equations of unknown origin. Artificial gravity isn't anything like gravity, they say, and who knows how it might affect humans over the long term.

It therefore seems worthwhile to step back and develop an argument for artificial gravity beginning with the most fundamental principles of physics, with some supporting mathematics to demystify the equations, followed by a brief review of research on the human tolerance for rotation, and a sampling of engineering studies that address various technical challenges.

The fundamental physics and math are old and reliable, but not often addressed in discussions of human space habitation. Along with the theory, there are ample practical studies of human factors, physiology, and engineering, conducted over the past several decades, to support artificial gravity as the most comprehensive countermeasure for weightlessness in long-duration space habitation.

II. Origin of the Concept and Terminology

"So if we want to get into the discussion of physiological countermeasures let's stop using terms coined by the popular press …"

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Konstantin Eduardovich Tsiolkovsky (Константин Эдуардович Циолковский) proposed and described artificial gravity more than a century ago – about the same time that he developed the Rocket Equation that still serves as a cornerstone of launch system engineering. He coined the term "artificial gravity" (or its Russian equivalent) long before the popular press got wind of it. But, if all that a person knows about artificial gravity is what they glean from the popular press, then they might well have cause to be suspicious of its scientific merit.

In 1903, Tsiolkovsky published "The Exploration of Space by Means of Jet Devices" («Исследование Мировых Пространств Реактивными Приборами») in the journal *Scientific Review* (*Научное Обозрение*), in which he developed the mathematical foundation of modern spacecraft engineering. The article includes an illustration of a large rotating artificial-gravity space habitat [Logsdon, Butler, 1985].

In 1896, Tsiolkovsky had already begun writing his visionary narrative *Beyond the Planet Earth* (*Вне Земли*) [Tsiolkovsky, 1960]. He completed it in 1916, and finally published it in 1920. Though thinly veiled as a work of fiction, *Beyond the Planet Earth* was a scientific discourse that expounded his revolutionary ideas on rocketry and space travel. As a work of literature, it reads like a story problem in Newtonian physics. Forty-five years before anyone had actually flown in space, Tsiolkovsky anticipated many of the problems associated with living and working in a weightless environment, including the loss of muscle tone and the need for foot and waist restraints at workstations. More to our purposes, he described how artificial gravity could be produced by rotating the habitat, and he anticipated the significance of the radius of rotation and the relative movement of people within the habitat:

> Most of the men had felt nothing at all, particularly when the radius of rotation was large. But in the case of a man's moving rapidly and independently, the artificial gravity created by centrifugal force produced a very interesting effect, which we may have occasion to describe later.

III. Theoretical and Mathematical Basis

*"And the countermeasures we're talking about here, let's be clear: are *not* 'artificial gravity' nor are they gravity of any kind."*

—

"I do not accept the premise of artificial gravity. G propagates at the speed of light. It has certain noted effects on the physiology of (most) living things. 1 or 10 RPM over any moment arm does not even begin to simulate the force in question …"

A. The Fundamental Forces of Physics

The material in this section is mostly textbook physics, though it tends to be spread over multiple chapters. Davies [1989] provides a good overview. Wikipedia [2016] also offers handy articles on various aspects.

Modern physics posits Four Fundamental Forces or "interactions": strong nuclear, weak nuclear, electromagnetic, and gravitational. Every interaction in the physical universe is ultimately reducible and attributable to these Four. Among these, gravity is the weakest; it also stands apart as the only one that doesn't conform to the Standard Model of particle physics. Each of the other three operates through some mediating particle: the electromagnetic interaction through photons; the weak nuclear interaction through W and Z bosons; and the strong nuclear interaction through gluons.

Some physicists hypothesize a graviton to bring gravity into line with the others and arrive at a Theory of Everything, but the theory remains incomplete and unsupported by experimental evidence. In Einstein's General Theory of Relativity, the apparent gravitational force is a consequence of the curvature of four-dimensional space-time.

G does not propagate. Pardon the pedantry, but we must strive for precision in discussing fundamental theories. The uppercase symbol G represents a constant coefficient in Newton's Law of Universal Gravitation and in Einstein's General Theory of Relativity. It relates fundamental units of distance, mass, and time (and derived units of force and acceleration). In SI units, its value is approximately $G \approx 6.674 \times 10^{-11} \cdot m^3 \cdot kg^{-1} \cdot s^{-2}$, subject to experimental refinement. The lowercase symbol g is a measure of the apparent gravitational acceleration at the surface of the Earth. Though the actual value varies slightly with latitude, altitude, and other factors, the 3rd General Conference on Weights and Measures in 1901 defined its standard value as $g = 9.80665 \cdot m \cdot s^{-2}$ [Bureau International des Poids et Mesures, 2006].

If by "G" one means "gravitational force," then that also does not propagate as the other three (quantum mechanical) forces do. Light, for example, propagates from luminous objects via continuous streams of photons. There is no evidence for an analogous graviton that streams between massive objects and "propagates at the speed of

light." Rather, it's the *change* in the gravitational field that propagates as wave-like fluctuations of space-time as massive objects move. When we observe the light from celestial objects across light-years of space-time, we see them as they were, where they were, years go. Any observable gravitational effect also emanates from the same location. We can't use gravity to determine where the object has moved to "now" versus where it appears to be in the past. The recent confirmation of gravitational waves is the latest in a long chain of experimental results that verify Einstein's General Theory of Relativity.

The General Theory of Relativity (GTR) conceives gravity not as a force like the others, but rather as a curvature of space-time. Moreover, it holds that gravity and acceleration are equivalent, differing only as a matter of the frame of reference. In a chapter titled "The Equality of Inertial and Gravitational Mass as an Argument for the General Postulate of Relativity," Einstein proposed a thought experiment: In a large region of empty space, far removed from any appreciable mass, a large chest containing an observer is accelerated upward by a "being." Every experiment the observer can perform within the confines of the chest indicates that the chest is suspended motionless in a gravitational field. Einstein concluded that: "a gravitational field exists for the man in the chest, despite the fact that there was no such field for the coordinate system first chosen" [Einstein, 1961]. So, if "artificial gravity" is a misnomer for acceleration, it's not because it's not gravity, but rather because it's not artificial. According to GTR, gravity and acceleration are interchangeable according to the frame of reference.

> *"... theorize all you like. Einstein was never exactly interested in making immediate problems doable."*

Einstein provided the theory. It has passed every test presented to it. If designers, planners, and policy makers fail to put his theory into practice "to make immediate problems doable," that's no fault of Einstein. There's no reason to suppose that he was never interested in seeing practical applications. (I think Einstein would be tickled to see how the application of the GTR enables the precision of the GPS (Global Positioning System) that has become a staple of modern life.)

B. The Fundamental Forces and Physiology

Whatever gravity may or may not be (a force like the other three Fundamentals, or an illusion wrought by the curvature of space-time), it's mostly irrelevant to human health, whether on or off the Earth. All mechanical and chemical phenomena – including *bio*mechanical and *bio*chemical – are consequences of the electromagnetic interaction between atoms. Gravity is relevant only insofar as it draws atoms close enough together for electromagnetic interaction.

Consider astronauts in low Earth orbit. At 400 km above the surface of the planet, the intensity of Earth's gravitational field is about 89% of the surface value. According to Newton's Law of Gravitation, the attraction is inversely proportional to the square of the separation distance. Taking the radius of the Earth as 6,371 km, the orbital proportion is: $(6{,}371/(6{,}371+400))^2$. Yet the astronauts are weightless and suffer all of the physiological consequences of that. That is *not* because some mysterious outward *centrifugal* force cancels gravity. If anything canceled gravity, astronauts wouldn't orbit; they would drift away on straight-line tangents. The very existence of the orbit is evidence of a substantial unopposed inward-acting *centripetal* gravitational field. Stop the orbit in its track, stop the circular motion and let the astronauts fall straight toward the planet, and they'll experience precisely the same weightlessness as when orbiting, until they impact the atmosphere. Centrifugal force has nothing to do with it.

Consider passengers on an airplane flying parabolas in the Earth's atmosphere, a mere 10 km above the surface. As the plane goes ballistic and rounds the top of a parabola, the passengers experience weightlessness, even though they're in essentially the full intensity of Earth-surface gravity. This is not merely a simulation of weightlessness; this is precisely the same phenomenon that orbiting astronauts experience, the only difference being the short duration before the airplane either pulls out of the free-fall or impacts the Earth.

Consider drop tubes such as the one at the Zero Gravity Research Facility at the NASA Glenn Research Center. For the 5 seconds it takes a payload to drop 140 m to the bottom, it's weightless, despite its continual immersion in the full intensity of Earth's gravitational field.

Astronauts in orbit don't lack gravity. There must be something else that we have on the Earth's surface, and they lack, that keeps us healthy. That something is acceleration by another force – specifically, electromagnetic (mechanical) force. It is the upward acceleration of the floor, not the downward acceleration of gravity, that provides us with weight. Whether that upward push arises as a reaction to gravity, or is provided by rocket thrust, structural tension, or some other mechanical means, is irrelevant. It seems almost *as if* inertial space itself draws

into massive objects, and electromagnetic force between colliding atomic shells accelerates atoms against that, in accordance with Newton's Second Law of Motion: force equals mass times acceleration.

Acceleration is the key. Vice-like compression pushing equally from opposite directions, such as provided by "penguin suits," loads the skeletal system but has no effect on pressure gradients in fluids or soft tissues. Mechanical acceleration propagates electromagnetic force throughout the body's material substance exactly as if it were suspended in a gravitational field.

C. Position, Velocity, Acceleration, Force, Energy, and Rotation

Position, velocity, and acceleration are vector quantities, having both a magnitude and a direction measured with respect to some coordinate system. Velocity is the rate of change of position over time: $\mathbf{V} = \dot{\mathbf{R}}$. Acceleration is the rate of change of velocity over time: $\mathbf{A} = \dot{\mathbf{V}} = \ddot{\mathbf{R}}$.

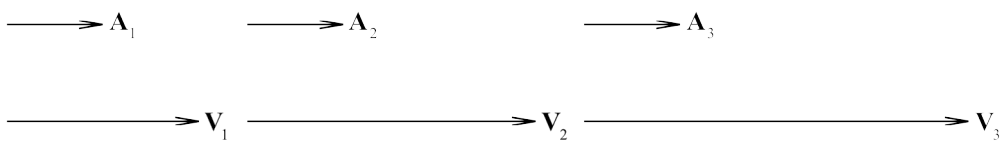

Figure 1. Linear acceleration.

According to Newton's Second Law of Motion, acceleration requires the application of a force in proportion to mass: $\mathbf{F} = m\mathbf{A}$. So, force is also a vector, aligned with and proportional to acceleration. The application of a force over any displacement requires work. The work done in accelerating a body manifests as a change in its kinetic energy.

Linear acceleration, depicted in Figure 1, is always parallel to the velocity. It modifies the magnitude of the velocity, but not its direction. Because the force and velocity are aligned, it requires an ever-increasing energy input. Not only the energy, but even the power (the *rate* of energy input) must continually increase relative to an inertial frame. It also precludes an object from staying in any particular vicinity for very long. With the right intensity over a useful trajectory, it would be a perfect substitute for planetary gravity, if only it were sustainable.

Centripetal acceleration, depicted in Figure 2, is always perpendicular to the velocity. It modifies the direction of the velocity, but not its magnitude. Because the force and velocity have no alignment (the projection of one onto the other is zero), the force does no work and consumes no energy. Once the rotation is brought up to some target value of rotations per minute (rpm), it's self-sustaining through the conservation of energy and momentum. Rotating structures can also be placed in stable planetary orbits. This makes centripetal acceleration the only viable means of providing weight in space, apart from a planetary surface, for long durations.

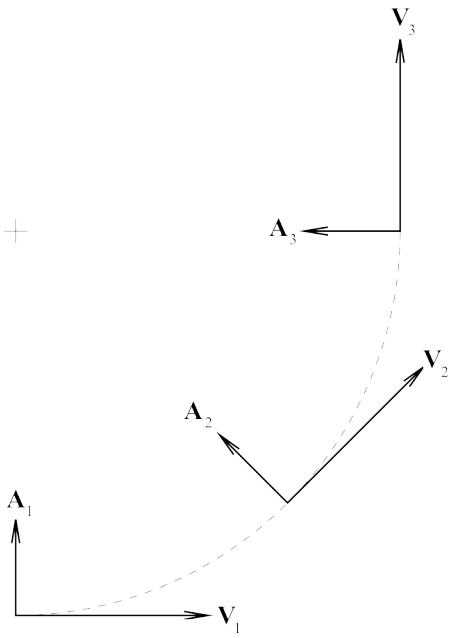

Figure 2. Centripetal acceleration.

Life in rotating environments is subject to certain "very interesting effects," as Tsiolkovsky put it. Figure 3 shows the path of a falling particle in a rotating habitat as seen from an inertial (non-rotating) reference, and Figure 4 shows the path as seen by a rotating inhabitant. There is neither a centrifugal force that pushes the particle toward the floor, nor a Coriolis force that diverts it. The apparent curvature of the particle's path is an illusion wrought by the rotation of the inhabitant. The only operative forces are the mechanical (electromagnetic) forces that propagate from contact with structural elements: centripetal force exerted by the floor; and Coriolis force – in this case, exerted by radial structures such as pipes, ladders, or elevator shafts – that would *constrain* the particle's path *against* the apparent curvature. The degree of curvature depends on the ratio of the particle's initial "height" (measured inward from the floor) over the radius of the floor: the bigger the ratio h/R_f, the greater the apparent curvature. Figures 3

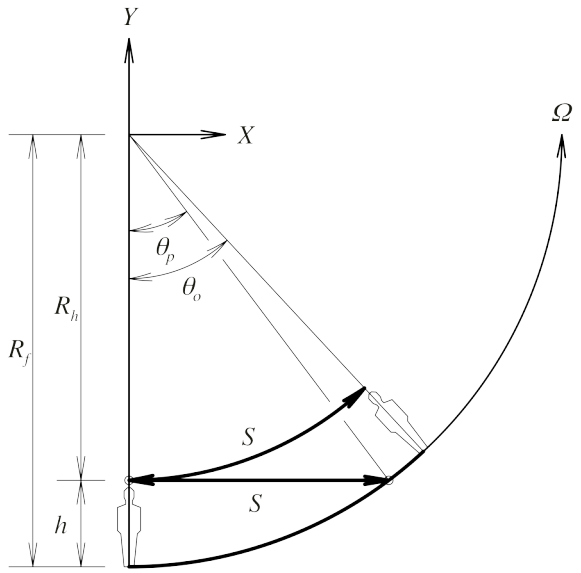

Figure 3. Inertial view of a dropping particle in a rotating habitat. The particle travels a tangential distance *S* with constant velocity from its initial position above the floor (on the left), until it strikes the floor (toward the right). If it had not been dropped, it would have travelled the same distance *S* along an arc, along with the observer.

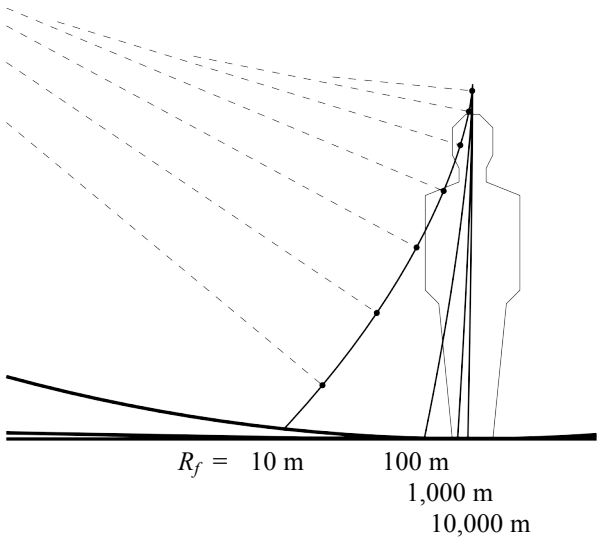

Figure 4. Rotating view of a dropping particle. The dotted lines show the particle's inertial linear path, which seems to rotate in the observer's rotating frame of reference. The particle appears to follow an involute curve, with a curvature dictated by the ratio of the initial height *h* over the floor radius R_f. Smaller ratios yield less curvature.

and 4 show an extreme condition, with an initial drop height of 2 m and a floor radius of only 10 m, to illustrate the effect.

Centripetal and Coriolis acceleration are purely mathematical concepts, independent of the cause, purpose, or mechanism of the acceleration. The equations that describe them are not merely "best fit" expressions for observed behavior of particles in rotating systems. Rather, they are necessary mathematical consequences of the very definitions of position, velocity, acceleration, and rotation. The details of their derivation are not crucial to the thesis of this paper. Suffice to say that they're the outcome of straightforward though perhaps tedious application of the principles of trigonometry and calculus, deferred to the Appendix. The causative forces are not mysterious "fifth forces" that arise from the ether merely because the frame is rotating; they are commonplace mechanical forces that abide by Newton's Second Law of Motion: force equals mass time acceleration.

IV. Physiology and Adaptation

"… all kinds of issues about the value v risks of substituting centrifugal / centripetal force for gravity. Those forces wreak havoc on the neurovestibular system and are not demonstrably useful as a countermeasure. Like many forms of chemotherapy, we risk making the subject much worse and weakening them for the fight [sic] *in the course of stabbing more or less in the dark about the root cause of musculoskeletal degeneration in a low-g environment."*

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"We should not call all sorts of dynamic-loading countermeasures equivalent to artificial gravity. I think anything less than a certain large radii (who knows what that may be – 300 ft?) should fall outside of the "artificial gravity" definition and just be called a vomit centrifuge. Even 300 ft. would make most people queasy. Serious artificial gravity research should be capable of radii over 700 ft. Otherwise why even bother – it would just be another vomit centrifuge."

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"… if it rotates faster than 1 rpm, the majority of human beings tested upchuck. (Even at 1 rpm, a significant fraction of the people tested upchuck; you need to get the rotation rate down to about 0.25 rpm in order for the general population to not upchuck.)"

A. Weightlessness

It must be noted that the alternative to "artificial gravity" in space is not "natural gravity," but rather weightlessness. Weightlessness wreaks havoc on the neurovestibular and other systems and is demonstrably unhealthy. Astronauts currently endure one to three days of "space adaptation syndrome" – including nausea, vomiting, and lethargy – as their vestibular systems adjust. There's evidence that the brain comes to rely more on visual cues and less on vestibular senses of motion or position [Connors, Harrison, Akins, 1985; Covault, 1983; Merz, 1986].

Current countermeasures to musculoskeletal degeneration and other ailments, that rely on diet and medication, are essentially chemotherapies that address only individual symptoms of weightlessness, not the root cause, and run the risk of unintended side effects.

For example, adding calcium to the diet to avert bone demineralization increases the risk of developing urinary stones. The bones are unable to retain the calcium they already have, and levels in the blood are consequently already elevated [Connors, Harrison, Akins, 1985; Oberg, Oberg, 1986]. According to Wolff's Law, bone is deposited where needed and resorbed where not needed. The leading theory for what causes this is a piezoelectric effect of bone tissue under stress [Chaffin, Andersson, 1984; Mohler, 1962; Woodard, Oberg, 1984]. Take away the stress; take away the bone.

Muscle also changes its mass and structure when not stressed [Connors, Harrison, Akins, 1985; Merz, 1986]. Many of the other ill effects of extended weightlessness are elements of a cascade failure triggered by the loss of a fluid pressure gradient. The shift of fluids from the legs toward the torso and head provokes changes in cardiac size, fluid loss, red blood cell loss, electrolyte imbalances, and other undesirable adaptations [Connors, Harrison, Akins, 1985; Gunby, 1986; Marwick, 1986; Merz, 1986; Oberg, Oberg, 1986; Woodard, Oberg, 1984].

Current countermeasures reduce the rate of bone and muscle loss but don't halt it. There is currently no countermeasure for the visual impairment / intracranial pressure (VIIP) syndrome [Alexander et al., 2012]. According to Schmidt, Goodwin, and Pelligra [2016]:

> The incompletely defined, but inevitable, adverse effects of 30-months exposure to reduced gravity during a Mars mission (0 g during transit and 0.38 g while on the planetary surface) are not likely to be remedied by exercise, pharmaceuticals, or a combination of both [Paloski et al. 2014]. The reasons for their failure are predictable.

B. Rotation

The centripetal and Coriolis forces that impinge on bodies in rotating structures are not of a different nature than the force that gives us weight on the surface of the Earth. They're mechanical forces that propagate through bodies via the electromagnetic interaction of atoms bumping together. The Earth also rotates, so even stationary Earth-dwellers are subject to miniscule centripetal and Coriolis forces associated with that. The difference for rotating space structures, several orders of magnitude smaller and faster than the Earth, is a matter of quantity and thresholds, not of essence. Earth dwellers are accustomed to a very uniform gravitational field and may need time to adapt to the less uniform field offered by rotating space habitats.

Although we may not know all of the nanoscale dependencies of human physiology on weight, we do know that they're ultimately electromagnetic in nature, and that mechanical acceleration propagates weight through bodies via those same electromagnetic forces. Schmidt, Goodwin, and Pelligra [2016], who study the effects of gravity, weightlessness, and centripetal force on biological systems at the molecular level, confirm this:

> It is fortuitous for future space voyagers that, in accordance with Einstein's "Theory of Equivalence," the human body cannot distinguish between the effects of accelerations generated by gravitation or by centrifugation (though effects of the Coriolis force must be considered). It responds identically to both, at the cellular, systemic, and behavioral levels.

The question is not *whether* "artificial gravity" works, but rather the details of *how* gravity and acceleration work, and the parameters, dosages, and response thresholds for an effective countermeasure to weightlessness.

Experiments on smaller mammals and tissue samples have demonstrated the potential health benefits of centripetal acceleration in space, as well as in ground-based analogs. On the Soviet satellite Cosmos 936 in 1977, the life span of rats exposed to centrifugation during 18.5 days of space flight was significantly greater than that of non-centrifuged control animals. Centrifugation reduced red blood cell loss and preserved bone minerals, structure, and mechanical properties [Connors, Harrison, Akins, 1985]. However, there were some adverse effects from the very high angular velocity of 53.5 rpm, including altered equilibrium, righting reflex, and orientation disorders [Schmidt, Goodwin, Pelligra, 2016]. On Spacelab D-1 in 1985, T-cell function – which is severely hampered in

micro gravity – was preserved in artificial gravity via centrifugation [Diamandis, 1987]. Clément, Charles, Norsk, and Paloski [2015] summarize evidence from other space-based and ground-based experiments.

In the early years of manned spaceflight, before Skylab and Salyut, there was considerable doubt that humans could survive long-duration weightlessness. Researchers at the Naval Aviation Medical Acceleration Laboratory (Johnsville, Pennsylvania), the Naval Aerospace Medical Research Laboratory (Pensacola, Florida), and the NASA Langley Research Center, conducted experiments with human subjects in centrifuges and rotating rooms, under continuous acceleration for up to several days, to study their adaptation to rotation. Graybiel [1977] gave a succinct summary of his findings:

> In brief, at 1.0 rpm even highly susceptible subjects were symptom-free, or nearly so. At 3.0 rpm subjects experienced symptoms but were not significantly handicapped. At 5.4 rpm, only subjects with low susceptibility performed well and by the second day were almost free from symptoms. At 10 rpm, however, adaptation presented a challenging but interesting problem. Even pilots without a history of air sickness did not fully adapt in a period of twelve days.

The sample size for these experiments was small, as is the sample size for all of human spaceflight. Nevertheless, it appears that Graybiel's subjects had no more difficulty adapting to rotation at 3.0 rpm than astronauts have adapting to weightlessness, and even 5.4 rpm might be comparable.

Based on similar research programs, Hill and Schnitzer [1962], Gilruth [1969], Gordon and Gervais [1969], Stone [1973], and Cramer [1985] published comfort charts to delineate the boundaries of human adaptation to rotation. Though their estimates varied, they all agreed that 2 rpm is comfortable; Gilruth even characterized everything below 2 rpm as "optimum comfort." Several of them estimated the upper limit as high as 6 rpm. Figure 5 is a composite of those charts. The green central zone depicts conditions that all agreed are comfortable; the red periphery depicts conditions that all agreed are uncomfortable; the hues ranging through yellow and orange depict regions of disagreement and conditions that probably require some adaptation to achieve comfort.

Table 1 summarizes some of the estimated boundary points. For a target effective centripetal acceleration (A), Table 1 shows the minimum tangential velocity (V) and radius (R), and maximum angular velocity (Ω). The original charts allowed for centripetal accelerations less than 1 g. Since those are not known to be sufficient countermeasures, Table 1 is more conservative in maintaining 1 g, except for Gilruth, who specified a maximum

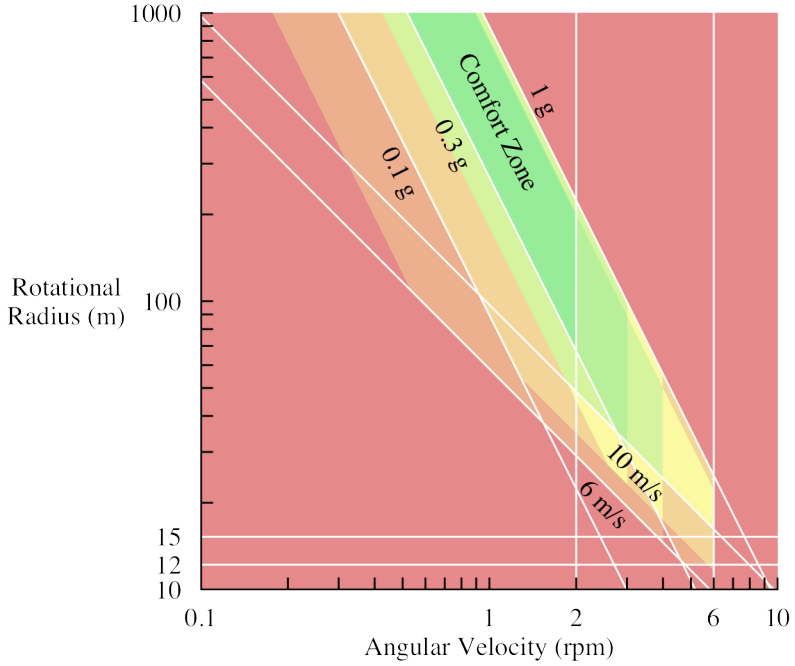

Figure 5. Composite comfort chart, based on Hill and Schnitzer [1962], Gilruth [1969], Gordon and Gervais [1969], Stone [1973], and Cramer [1985]. The green central zone depicts conditions that all agreed are comfortable; the red periphery depicts conditions that all agreed are uncomfortable; the hues ranging through yellow and orange depict regions of disagreement.

centripetal acceleration of 0.9 g. (He didn't specify a reason, but it might have been to allow for some additional Coriolis acceleration without exceeding a total of 1 g.) The boldface values are the limiting criteria; the others are calculated from them. Note that only the most conservative estimate, for Gilruth's "optimum comfort," exceeds a radius of 100 m (328 feet). At Cramer's limit of 3.0 rpm, for which Graybiel observed no difficulty adapting within 2 days, the radius is only 99.4 m. Increasing the angular velocity to 4.0 rpm decreases the radius to only 55.9 m. In comparison, the International Space Station measures 108.5 m × 72.8 m × 20 m.

Table 1: Target centripetal acceleration, minimum tangential velocity and radius, and maximum angular velocity, for "comfortable" rotation. The controlling parameters appear in boldface; the others derive from them.

Author	Target $\frac{A}{9.81}$ (g)	Min. V (m/s)	Min. R (m)	Max. $\frac{30\Omega}{\pi}$ (rpm)
Hill & Schnitzer [1962]	**1.00**	23.4	55.9	**4.0**
Gilruth [1969]	**0.90**	14.0	22.4	**6.0**
Gilruth "optimum" [1969]	**0.90**	42.1	201.2	**2.0**
Gordon & Gervais [1969]	**1.00**	15.6	24.8	**6.0**
Stone [1973]	**1.00**	14.7	22.1	**6.4**
Cramer [1985]	**1.00**	31.2	99.4	**3.0**

Researchers at Brandeis, MIT, and other facilities continue such studies. Extending those precedent findings, Lackner and DiZio [2003] claim an even higher upper limit for rotation:

> By contrast, our recent work has shown that sensory-motor adaptation to 10 rpm can be achieved relatively easily and quickly if subjects make the same movement repeatedly.

The chief limit to comfort in rotation at higher angular speeds is the onset of dizziness and illusions associated with cross-coupled rotations. This occurs when an inhabitant rotates his or her head about a local body axis that's not aligned with the rotation of the habitat. The result is a vestibular illusion of rotation around an axis perpendicular to the other two [Clark, Hardy, 1960; Lally, 1962]. For example: yawing the head left-right around a "vertical" axis, in a habitat that's simultaneously pitching around its own "north-south" axis, may produce an illusion of roll around the "east-west" axis. This phenomenon is explained mathematically, mechanically, and medically, by Euler's equations of rotational motion (which also explain and predict the precession of rotating tops). The cross-product of the angular velocities yields an angular acceleration around a mutually-perpendicular axis that stimulates the semicircular canals of the inner ear *as if* they were rotating around that axis. The mismatch of visual and vestibular senses of motion leads to motion sickness [Connors, Harrison, Akins, 1985; Merz, 1986]. This is not fundamentally different from what sailors on wavy waters have encountered for millennia, except that in the rotating habitat the phenomenon is far more predictable, controllable, and amenable to adaptation. Habitat designers can and should plan for the phenomenon and its effects, to aid the inhabitants' adaptation – for example, by orienting tasks to avoid cross-coupling rotations, and by providing visual cues to keep inhabitants oriented to the habitat's rotation [Hall, 1995; Hall, 2006; Ramsey, 1971].

V. Engineering and Design Studies

A. Engineering Studies

Accommodating artificial gravity undoubtedly adds complexity to spacecraft design. Yet, those who object that it's too much seem to underestimate or ignore the complexity of the alternative: myriad physiological damages from chronic weightlessness and disparate insufficient countermeasures. Much of the perceived complexity of artificial gravity may stem from fear of the unknown. Many preliminary studies have begun to explore the solution space. What follows is but a small sample.

At the third Case for Mars conference in 1987, Schultz, Rupp, Hajos, and Butler [1989] summarized a NASA multicenter study for an artificial-gravity crew transport vehicle as part of a split mission to Mars, with a round-trip time of 420 days. They examined various options for module arrangements, mass distribution, and structural

systems, but adopted a fairly conservative rotation rate of 2 rpm, which for 1 g acceleration would require a habitat rotational radius of 224 m. They estimated that their artificial gravity design would incur a 26% increase in mass and a 10% increase in cost versus a zero-gravity vehicle in the same mission scenario.

Fifteen years later, another NASA multicenter study arrived at a significantly different conclusion, estimating only about 1/5 the percentage mass increase of the earlier study. In 2002, the NASA Exploration Analysis and Integration Office organized an extensive assessment of artificial gravity impacts on deep-space vehicle design [Joosten, 2007]. Though the investigators initially aimed for a mission-independent assessment, for trade studies they ultimately adopted an opposition-class split-mission Mars transfer scenario. They considered multiple facets of the vehicle design, but specifically highlighted the impact of artificial gravity on the choice of propulsion system. Rather than tacking artificial gravity onto a system with an *a-priori* selection of incompatible propulsion, this study started with artificial gravity as the prime requirement, then chose low-thrust nuclear electric propulsion (NEP) as the most compatible. They also adopted a less conservative rotation rate of 4 rpm, bringing the 1-g radius down to 56 m. According to Joosten [2007]:

> Results from this study produced a unique approach to vehicle steering and attitude control without massive, despun vehicle components or excessive propellant consumption. Very little (~5%) additional structural or propellant mass was identified above that required for zero-gravity transfer.

Cables or tethers might offer the most compact and least massive structural support for artificial gravity at a large radius and slow rotation, but suffer from a perceived incompatibility with vehicle thrust and maneuverability. To address those concerns, Landau [2008] has performed a detailed mathematical analysis of the dynamics for a tethered artificial-gravity Mars transit vehicle. His proposal allows for high-thrust propulsion and maneuvering without the need to halt the rotation or reconfigure the vehicle.

Carroll [2010] has examined structural design, mass fractions, rendezvous and berthing strategies, photovoltaic solar cell placement, orbital characteristics, atmospheric drag, and reboost implications for four "dumbbell" configurations of a low Earth orbital (LEO) artificial-gravity research facility. Each configuration would provide Lunar-intensity artificial gravity at one node and Martian-intensity at the opposite node, with a rotation rate between 0.25 and 2 rpm and a corresponding facility length ranging approximately from 120 m to 8 km. The connecting structure would comprise a combination of rigid modules, air-beam tunnels, and cables, depending on the length. The calculations are preliminary estimates appropriate to initial trade studies, but cover a broad span of spacecraft design parameters. It's not known whether Lunar or even Martian-intensity gravity is sufficient to preserve human health. One purpose of this facility would be to examine that issue, as well as human tolerance for various rotation parameters, before committing humans to Mars missions of unprecedented duration.

Sorensen [2005] has also studied the design of a tether-based variable-gravity research facility. He describes a technique for starting and stopping the rotation without expending any propellant, by means of gravity gradients and reeling, preserving angular momentum but varying the spacecraft's moment of inertia. Moreover, he cites a mechanical innovation known as a Canfield Joint that enables the pointing of solar collectors, communication antennas, and even thrusters, from the spinning structure, without the need for despinning interfaces.

Jevtovic [2015] has analyzed an electromagnetic system for rotating a spacecraft without propellant, using proven technology derived from magnetic-levitation trains. The rotation mechanism is contactless and frictionless and powered by solar collectors.

Sullivan [2002, 2003] has developed a uniquely asymmetric design for an electrically rotated artificial-gravity spacecraft in which the power generation and motor assembly also serves as the countermass for the habitat. The spacecraft's overall center of mass and rotation axis lie between the spinning countermass and the habitat. The rotation of the uninhabited countermass isn't constrained by issues of human comfort and can be much faster than the spacecraft's overall rotation.

B. Habitat Design

Engineers have devoted many hours and pages of analysis to the design of artificial-gravity spacecraft, examining mass distributions, moments of inertia, structural stresses, rotating interfaces, and the dynamics of starting, stopping, and steering the rotation. Yet the design of the habitat itself has often been neglected or defaulted to "Earth-normal" gravitational concepts.

As a case in point, consider again the 2002 NASA study referenced earlier [Joosten, 2007]. Only the last of its 17 figures – a stack of 3 small floor plans – reveal anything about the habitat's internal arrangement and the inhabitants' living conditions. The three paragraphs devoted to "Human Factors and Habitability" say nothing specific about the plans or accommodations for artificial gravity. The gist is that the "power-rich" 1-g environment

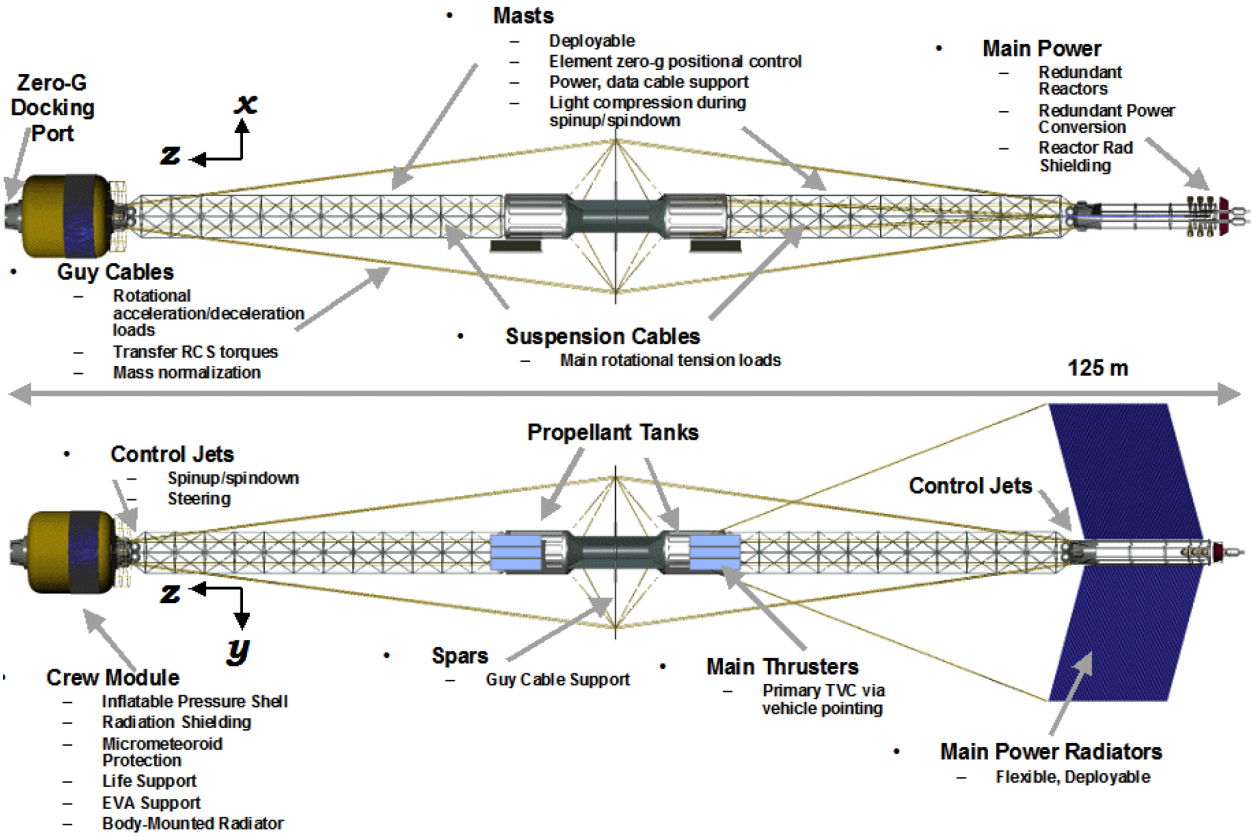

Figure 6. Spacecraft configuration proposed by the NASA Preliminary Assessment of Artificial Gravity Impacts to Deep-Space Vehicle Design (from figure 4 of Joosten [2007]).

should permit the adoption of familiar Earthlike furnishings and appliances, such as beds, chairs, a washer, dryer, and convection oven. This concept carried forward from an earlier "Phase I" study, included in the NASA report as "Attachment 2." The figure appears at a larger scale as "Appendix C" of that attachment, with essentially the same brief description as the final report.

Considering that artificial gravity was the driver behind every other detail of the spacecraft configuration, including the selection of propulsion and structural systems, it seems worthwhile to devote more attention to the *raison d'être* for artificial gravity in the first place: the habitat. Figure 6 shows the overall spacecraft configuration, with a TransHab-derived habitat module at the left. Figure 7 shows the habitat floor plans. (The slight distortion of the circular plans is present in the original document. The original also places the lower floor at the top of the figure; Figure 7 here reverses the stacking order compared to the original.) Let us dissect these plans.

The first thing that ought to be apparent, but is completely lacking, is any indication of the direction of rotation. This is critical, because it determines the orientation of the Coriolis forces, cross-coupled rotations, and almost everything peculiar about artificial gravity that requires the crew to adapt. Closer inspection reveals a ladder in the "south-east" quadrant of the Third and Second Floor plans, but the "north-east" quadrant of the First Floor plan; the drawings aren't consistently oriented. The pass-through from the Third to Second floor is on one side of the ladder, whereas the pass-through from the Second to the First Floor is on the other side. The designer might have considered this to be a safety "feature" that would limit the distance of a fall from the Third Floor. (The text offers no explanation.) But in fact, it's a "bug": by ignoring the orientation of the Coriolis force between the ladder and the climber, and its dependence on the direction of travel (ascending versus descending), it actually increases the likelihood of a fall. Beyond the ladder, the general circular symmetry of the plan exhibits a disregard for the crew's encounters with Coriolis accelerations and cross-coupled rotations. This is puzzling because it's so inconsistent with the attention devoted to these dynamics in all other aspects of the spacecraft configuration. We can't tell from the plans whether any crew activities are optimally oriented, but we can tell certainly that most are not, because they're oriented in all different directions. There are opportunities for essentially zero-cost improvements in these plans, without revising the basic premise of the module structure or its inventory of equipment and furnishings.

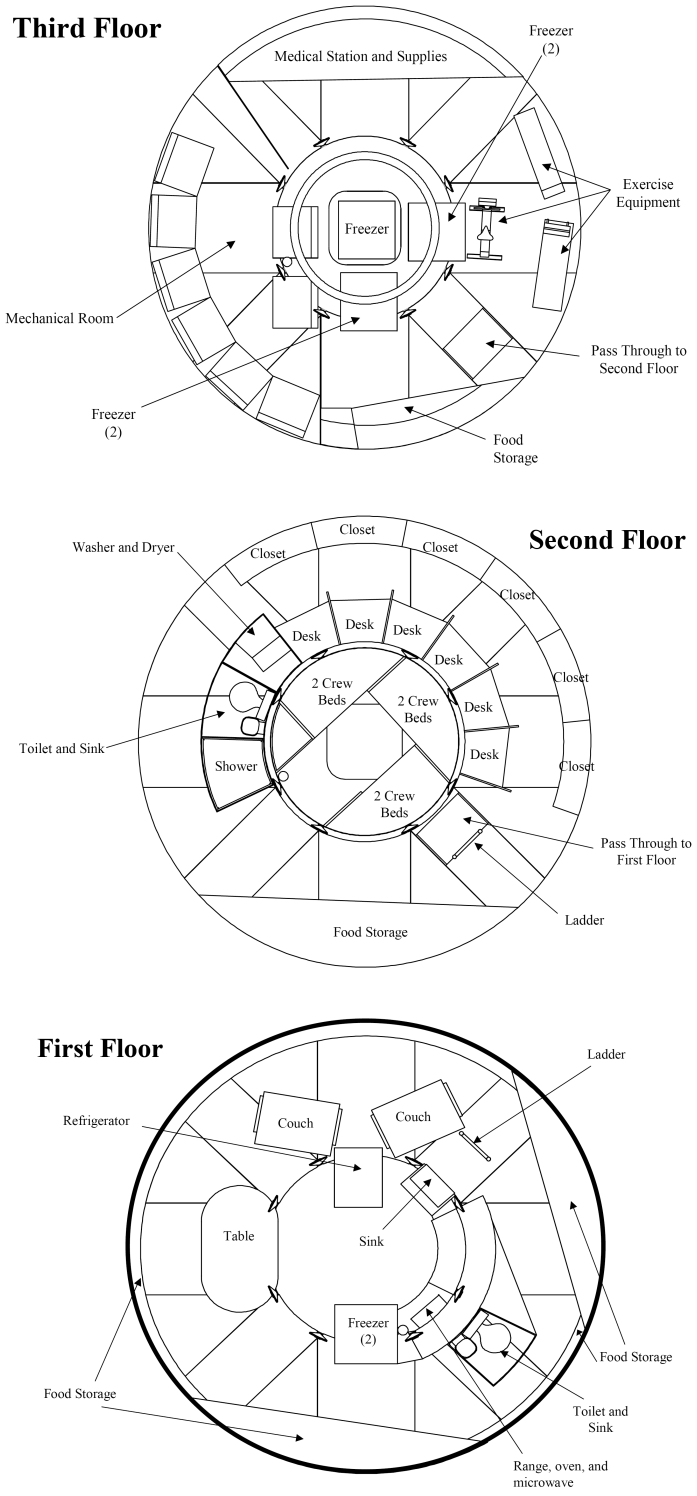

Figure 7. Habitat floor plans proposed by the NASA Preliminary Assessment of Artificial Gravity Impacts to Deep-Space Vehicle Design (from figure 17 of Joosten [2007]; the slight distortion of the circular plans is present in the original publication). These plans show no adaptation to the peculiarities of rotationally-induced artificial gravity.

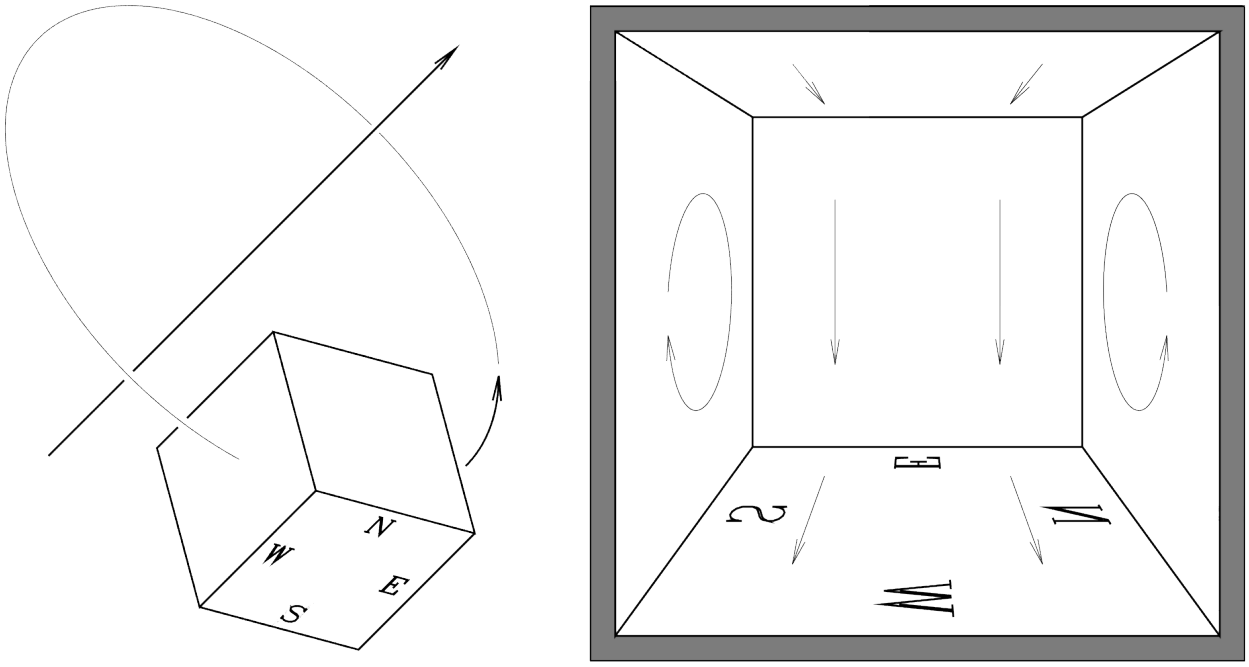

Figure 8. The cardinal directions (North, South, East, West) in artificial gravity. The arrows in (b) indicate the apparent rotation of the star field from the rotating point of view. Celestial objects set in the east and rise in the west.

The design advice that follows is largely conjecture, but is rooted in math, physics, physiology, and general principles of architectural design. The first such habitat will inevitably be an experiment. Nevertheless, it's vital to strive to get the details right the first time. Figure 10 depicts a partial revision of the plans in Figure 7 based on these suggestions.

When drawing plans for an artificial-gravity habitat, the first thing "on paper" should be a big arrow to indicate the direction of rotation, labeled with the rotation parameters. This should be explicit, highly visible, and in the designer's consciousness throughout the design process. All of the Coriolis accelerations occur in the vertical plane through that arrow (and planes parallel to it). To assign familiar cardinal orientations, call the direction of the tangential velocity "East" and its opposite "West." Use the right-hand rule for rotations to define "North" as the direction of the structure's angular velocity vector, and "South" as its opposite. Note that, compared to the convex surfaces of planets, in the concave landscape of artificial gravity (where the axis is overhead rather than underfoot), "North" and "South" have opposite handedness relative to "East" and "West," as shown in Figure 8.

Impose the north-south-east-west grid on the plan, and adhere to it in the layout of all crew activities, regardless of the profile of the habitat pressure vessel. Though planetary gravity is unbiased relative to the cardinal directions, rotationally induced artificial gravity is not. The pressure vessel does not insulate the crew from Coriolis accelerations, regardless of its shape. If imposing a rectilinear layout on a circular floor seems awkward, reconsider the orientation of the pressure vessel to avoid circular floors.

The most gravitationally critical element of this habitat is the ladder. Locate it first. Its plane should be parallel to the north-south axis so that Coriolis accelerations (on the east-west axis) are perpendicular to it. Climbers should ascend on the west side and descend on the east side, so that the Coriolis acceleration presses the ladder against the climber rather than pulling it away. The Coriolis force, added vectorially to the centripetal force, will make the ladder seem to slope on a catenary arch, as diagrammed in Figure 9 [Hall, 1999]. Traversing the ladder on the wrong side would place the climber precariously on the underside of the arch.

There are two options for accommodating different orientations for ascent and descent. One is to place the ladder at the center of a pass-through that's twice as big – providing a floor opening on both its east and west sides. The other option, perhaps trivially more expensive but more efficient in floor area, is to provide separate ladders for

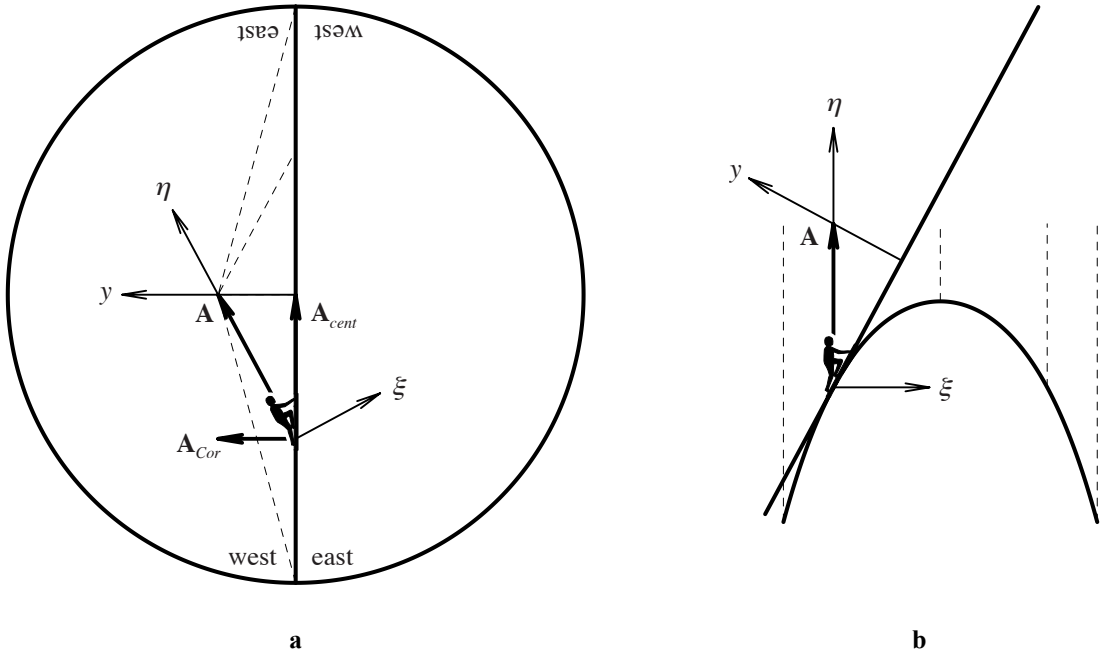

Figure 9. Climbing a ladder in artificial gravity. (a) Inertial view of the centripetal, Coriolis, and total acceleration vectors. (b) The apparent slope relative to the climber is a catenary arch. To stay above the curve, the climber must ascend on the west side of the ladder and descend on the east side.

ascent and descent on the east and west sides of the single pass-through. Climbers would ascend on the west side of the east ladder and descend on the east side of the west ladder.

If the ladder is offset from the center of the module, it should also be tilted slightly to align with the centripetal acceleration, which is always directed precisely toward the rotation axis (not necessarily parallel to the module axis or perpendicular to the floor). In any case, the designer should perform a vector analysis similar to Figure 9a to verify that the sum of the centripetal and Coriolis accelerations always presses the ladder against the climber.

Activities that demand the highest mobility should be located in the plan first, with the highest priority and attention to their orientation. Tasks that require frequent head pitch – such as alternately viewing a desktop work area and a vertical display – should be arranged so that the worker faces east or west (rather than north or south). In this orientation, the rotation of the head is parallel to the rotation of the structure and doesn't incur cross-coupled rotations. Task layouts that require frequent side-to-side yaw should be avoided since they will inevitably induce cross-coupled head rotations no matter which way they're oriented.

Areas that require the least mobility – such as relaxing, sleeping, and storage – are the least susceptible to Coriolis forces and cross-coupled rotations and may be arranged to fit the available area.

If the circular plan interferes with the proper orientation of activities relative to the rotation, then either the shape of the plan or the rotational radius must be reconsidered; for artificial gravity at several rotations per minute, the Coriolis effects are non-negotiable. The NASA study in question here assumed a TransHab module for the habitat. The original TransHab concept envisioned dividing the volume into three circular levels along the axis in a *weightless* environment [Kennedy, 1999]. It's ironic that this study, which criticized earlier ones for tacking artificial gravity onto incompatible propulsion systems, proceeded to tack it onto an incompatible habitat design. Adapting the habitat to artificial gravity requires more that just stiffening the floor structure.

The ideal orientation of the module for the artificial-gravity climate would be horizontal, rather than vertical, with its axis parallel to the axis of rotation. Motion parallel to that axis doesn't involve Coriolis acceleration. There would be straight walls on the east and west sides for workstations, allowing head pitch without cross-coupling with the spacecraft rotation. There would be perhaps two levels (or better, only one) instead of three, reducing the use of ladders and exposure to gravity gradients. Though the original TransHab structural design might not be well suited for a horizontal orientation, insofar as the module doesn't actually exist yet there's still opportunity to design a derivative better suited to the intended use. Reorienting the module cylinder from vertical to horizontal might have

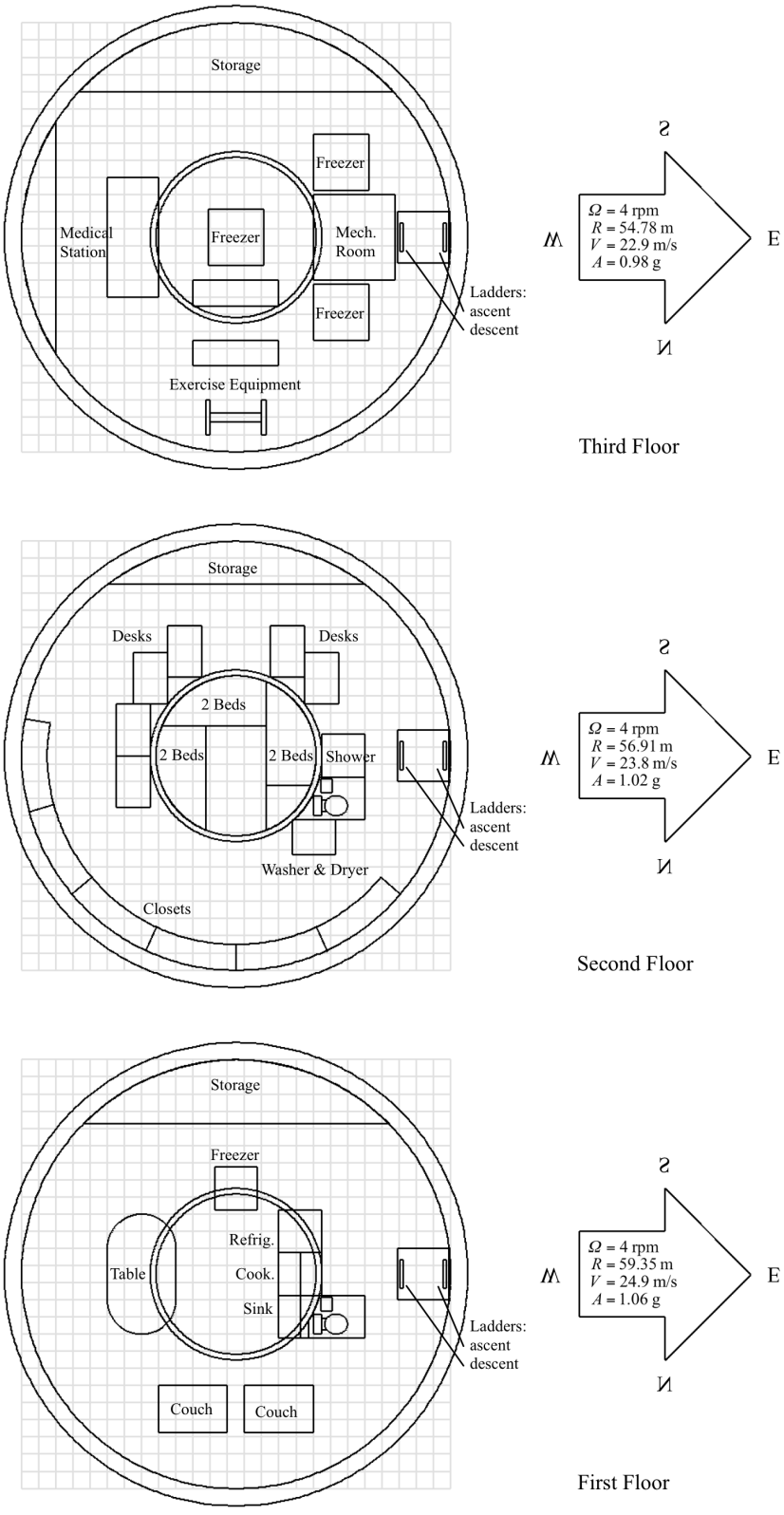

Figure 10. A suggested partial redesign of the habitat floor plans from Figure 7, with Coriolis accelerations in mind. The ladders should be sloped to align with the centripetal acceleration, accounting for their offset from the module axis. The circular plan is not ideal.

implications for the spacecraft's overall moment of inertia and stability. But, the cylinder is stubby, and it might not be so difficult to compensate by shifting other spacecraft masses, if crew comfort in artificial gravity were adequately prioritized.

In addition to reorienting the plan, the various wall, floor, and ceiling surfaces should vary in color, texture, and pattern in some consistent fashion to visually distinguish the cardinal directions – especially east and west. This may help the crewmembers to maintain their sense of orientation relative to the rotation and adjust their actions in anticipation of the inevitable Coriolis effects.

VI. Conclusion

Konstantin Tsiolkovsky first proposed and described artificial gravity more than a century ago. Yet after 55 years of crewed spaceflight, artificial gravity remains untried. We continue subjecting spacefarers to ever-longer durations of weightlessness. The repeated claim is that this is necessary to learn how to live in space. What we continue to learn is how unhealthy weightlessness is. What will it take to motivate a serious attempt to design, deploy, and test an artificial-gravity spacecraft? Will it take a loss of mission to finally jar space habitat design from its ossified complacency of doing things as they have always been done? The current attitude is analogous to the "normalization of deviance" that led to the *Challenger* and *Columbia* shuttle disasters [CAIB 2003; Vaughan 1996]. Astronauts aren't supposed to be losing bone mass, muscle mass, red blood cells, immune function, visual acuity, … it's not part of the design. But because these things have always happened, and haven't actually killed anybody yet, they're accepted as normal. Must we again await manifest proof of irreparable harm to the crew before we even begin to test alternatives to prolonged weightlessness?

The most complex system in any habitat is the inhabitant. Though biology and biochemistry have made great strides in recent decades, humans have yet to engineer any living organism from scratch, let alone a multicellular organism, let alone a vertebrate. In contrast, they've been engineering rotating interfaces since the invention of the potter's wheel more than five thousand years ago. Those who object that artificial gravity adds too much complexity to spacecraft design seem to underestimate the complexity of the alternative: continual vigilance for insidious effects of weightlessness, and continual search for other countermeasures to innumerable symptoms rather than addressing the common root of all the problems. As "unnatural" a living condition as artificial gravity may be, weightlessness is even more so.

Much of the perceived complexity of artificial gravity may stem from the prospect having to depart from established norms to explore new possibilities and establish new norms. Is that not at the heart of everything we aim to do in space? It's time for the next giant leap in space habitat design: artificial gravity.

Appendix: Mathematics of Centripetal and Coriolis Accelerations

The centripetal and Coriolis forces associated with rotation are not fundamentally different kinds of forces with unknown origins or effects. The formulas that describe them are not merely best fits for observed mysterious behaviors of particles in rotating systems. On the contrary, those observations confirm Newton's Second Law of Motion: force equals mass times acceleration. The expressions for the centripetal and Coriolis accelerations derive from the very definitions of rotation and acceleration. The derivation relies on principles of trigonometry and vector calculus.

The position of a point (x, y, z) can be described as a vector from some selected center or origin (0, 0, 0):

$$\begin{aligned} \mathbf{r} &= r_x \cdot \mathbf{i} + r_y \cdot \mathbf{j} + r_z \cdot \mathbf{k} \\ &= x \cdot \mathbf{i} + y \cdot \mathbf{j} + z \cdot \mathbf{k} \end{aligned} \tag{1}$$

where $\mathbf{r}$ is the position vector; $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are unit component vectors parallel to the x, y, and z axes; and r_x, r_y, and r_z are the projected lengths of $\mathbf{r}$ on those axes.

Velocity is the rate of change of position with time, denoted by a dot above the symbol:

$$\begin{aligned} \mathbf{v} = \dot{\mathbf{r}} &= \frac{d\mathbf{r}}{dt} \\ &= v_x \cdot \mathbf{i} + v_y \cdot \mathbf{j} + v_z \cdot \mathbf{k} \\ &= \dot{x} \cdot \mathbf{i} + \dot{y} \cdot \mathbf{j} + \dot{z} \cdot \mathbf{k} \end{aligned} \tag{2}$$

Acceleration is the rate of change of velocity:

$$\begin{aligned}\mathbf{a} = \dot{\mathbf{v}} = \ddot{\mathbf{r}} &= \frac{d^2\mathbf{r}}{dt^2} \\ &= a_x \cdot \mathbf{i} + a_y \cdot \mathbf{j} + a_z \cdot \mathbf{k} \\ &= \ddot{x} \cdot \mathbf{i} + \ddot{y} \cdot \mathbf{j} + \ddot{z} \cdot \mathbf{k}\end{aligned} \tag{3}$$

If the *xyz* coordinate system is rotated around its *z* axis by an angle θ relative to an *XYZ* system that shares the same origin and *z* = *Z* axis, then the coordinates of any point in the two systems have the following relationship:

$$\begin{aligned}X &= x \cdot \cos(\theta) - y \cdot \sin(\theta) \\ Y &= x \cdot \sin(\theta) + y \cdot \cos(\theta) \\ Z &= z\end{aligned} \tag{4}$$

To apply Newton's Second Law of Motion to evaluate the force acting on a particle, we need to determine the acceleration of the particle in an inertial, non-accelerated frame of reference. In the case of a rotating artificial-gravity spacecraft, let the *xyz* axes be tied to the rotating structure at *z* = 0, let the *XYZ* axes be non-rotating inertial axes with the same origin, and let the rotation be around their shared *z* = *Z* axis. The rotation angle θ is a function of the rotation rate Ω and time t:

$$\theta = \Omega \cdot t \tag{5}$$

Then the position of a particle expressed in the rotating and inertial reference frames is:

$$\begin{aligned}\mathbf{r} &= x \cdot \mathbf{i} + y \cdot \mathbf{j} + z \cdot \mathbf{k} \\ \mathbf{R} &= X \cdot \mathbf{I} + Y \cdot \mathbf{J} + Z \cdot \mathbf{K} \\ &= \left(x \cdot \cos(\Omega \cdot t) - y \cdot \sin(\Omega \cdot t)\right) \cdot \mathbf{I} + \left(x \cdot \sin(\Omega \cdot t) + y \cdot \cos(\Omega \cdot t)\right) \cdot \mathbf{J} + z \cdot \mathbf{K}\end{aligned} \tag{6}$$

where $\mathbf{I}$, $\mathbf{J}$, and $\mathbf{K}$ are unit component vectors parallel to the *X*, *Y*, and *Z* axes. The expressions for $\mathbf{r}$ and $\mathbf{R}$ represent the same position, but in different frames of reference. This is somewhat analogous to a temperature being expressed as 32° F = 0° C, or a distance as 1 ft = 0.3048 m, except that here the $\mathbf{ijk}$ reference is continually changing relative to the $\mathbf{IJK}$ reference.

The first and second derivatives of Eq. (6) yield the inertial velocity and acceleration of the particle. These rely on a few results from elementary calculus: the derivatives of the sine and cosine functions

$$\frac{d}{dt}\sin(t) = \cos(t) \quad ; \quad \frac{d}{dt}\cos(t) = -\sin(t) \tag{7}$$

the chain rule for functions of functions

$$\frac{d}{dt} f\left(g(t)\right) = \frac{df}{dg} \cdot \frac{dg}{dt} \tag{8}$$

and the rule for products of functions

$$\frac{d}{dt}\left(f(t) \cdot g(t)\right) = f(t) \cdot \frac{dg}{dt} + \frac{df}{dt} \cdot g(t) \tag{9}$$

Applying the rules of Eqs. (7-9) to Eq. (6) yields the inertial velocity of the particle:

$$\begin{aligned}
\mathbf{V} &= \dot{\mathbf{R}} \\
&= \left(-x\cdot\sin(\Omega\cdot t)\cdot\Omega + \dot{x}\cdot\cos(\Omega\cdot t) - y\cdot\cos(\Omega\cdot t)\cdot\Omega - \dot{y}\cdot\sin(\Omega\cdot t)\right)\cdot\mathbf{I} \\
&\quad + \left(x\cdot\cos(\Omega\cdot t)\cdot\Omega + \dot{x}\cdot\sin(\Omega\cdot t) - y\cdot\sin(\Omega\cdot t)\cdot\Omega + \dot{y}\cdot\cos(\Omega\cdot t)\right)\cdot\mathbf{J} \\
&\quad + \dot{z}\cdot\mathbf{K} \\
&= -\Omega\cdot\left(x\cdot\sin(\Omega\cdot t) + y\cdot\cos(\Omega\cdot t)\right)\cdot\mathbf{I} + \Omega\cdot\left(x\cdot\cos(\Omega\cdot t) - y\cdot\sin(\Omega\cdot t)\right)\cdot\mathbf{J} \\
&\quad + \left(\dot{x}\cdot\cos(\Omega\cdot t) - \dot{y}\cdot\sin(\Omega\cdot t)\right)\cdot\mathbf{I} + \left(\dot{x}\cdot\sin(\Omega\cdot t) + \dot{y}\cdot\cos(\Omega\cdot t)\right)\cdot\mathbf{J} + \dot{z}\cdot\mathbf{K}
\end{aligned} \tag{10}$$

There's a pattern in the plethora of terms in Eq. (10) that occurs so frequently that vector calculus provides an operator to encapsulate it: the cross-product. It has the form of a determinant of a 3×3 matrix comprising the basis vectors and the components of the vectors being crossed:

$$\begin{aligned}
\boldsymbol{\Omega}\times\mathbf{R} &= \begin{vmatrix} \mathbf{I} & \mathbf{J} & \mathbf{K} \\ \Omega_X & \Omega_Y & \Omega_Z \\ R_X & R_Y & R_Z \end{vmatrix} \\
&= \left(\Omega_Y\cdot R_Z - \Omega_Z\cdot R_Y\right)\cdot\mathbf{I} + \left(\Omega_Z\cdot R_X - \Omega_X\cdot R_Z\right)\cdot\mathbf{J} + \left(\Omega_X\cdot R_Y - \Omega_Y\cdot R_X\right)\cdot\mathbf{K}
\end{aligned} \tag{11}$$

The cross-product of two vectors is a third vector that's perpendicular to both of the operands, with a length proportional to the sine of the angle between them. The cross-product of parallel vectors (in the same or opposite directions) is the zero vector **0**.

Eq. (6) assumed a convenient coordinate system in which $\Omega_X = \Omega_Y = 0$, so several of the terms in the cross-product are zero. However, the pattern would apply for any arbitrary orientation of the rotation axis in the *XYZ* coordinate system. Eq. (10) can then be condensed to:

$$\mathbf{V} = \dot{\mathbf{R}} = \Omega\times\mathbf{R} + \mathbf{v} \tag{12}$$

The first term on the right, $\boldsymbol{\Omega}\times\mathbf{R}$, is the tangential velocity of the particle due to the rotation of the structure. The second term, $\mathbf{v}$, is the velocity of the particle relative to the rotating structure.

The next-to-last line of Eq. (10) is the cross-product, $\boldsymbol{\Omega}\times\mathbf{R}$. The last line of Eq. (10) expresses the value of $\mathbf{v}$ in components parallel to the **IJK** basis vectors – using Eq. (2), a rotation transform analogous to Eq. (4), and the instantaneous value of the rotation angle from Eq. (5), independent of the continual change of the angle. The cross-product accounts for the continual change of the angle.

The pattern in Eq. (12) is that the rate of change of the vector in the inertial *XYZ* system is equal to the cross-product of the angular velocity of the *xyz* system and the vector, plus the vector's rate of change in the rotating *xyz* system. This is known as the "Basic Kinematic Equation." It applies to any vector – not only for the change in position, but also for the change in velocity.

To find the inertial acceleration of the particle, we can apply the same pattern again:

$$\begin{aligned}
\mathbf{A} &= \dot{\mathbf{V}} = \ddot{\mathbf{R}} \\
&= \boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{R} + \mathbf{v}) + (\boldsymbol{\Omega}\times\mathbf{v} + \mathbf{a}) \\
&= \boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{R}) + 2\cdot\boldsymbol{\Omega}\times\mathbf{v} + \mathbf{a}
\end{aligned} \tag{13}$$

The first term on the right, $\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{R})$, is the centripetal acceleration. The second term, $2\cdot\boldsymbol{\Omega}\times\mathbf{v}$, is the Coriolis acceleration. The third term, $\mathbf{a}$, is the acceleration of the particle relative to the rotating system.

In the case of circumferential motion on the curved "floor" of a cylinder or torus, either prograde or antigrade, $\mathbf{a}$ itself is another centripetal acceleration:

$$\mathbf{a} = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \tag{14}$$

where $\boldsymbol{\omega}$ is the angular velocity relative to the rotating structure – e.g., walking speed divided by the radius of the arc: $\omega = v/r$.

If one isn't comfortable with the shortcut taken from Eq. (12) to Eq. (13), one can reapply the rules of Eqs. (7-9) to all of the terms in Eq. (10), recombine, and ultimately arrive at the same result.

Many authors negate the centripetal and Coriolis terms in Eq. (13) to express the "fictitious" or "imaginary" centrifugal and Coriolis forces as seen in the rotating reference *as if* it were inertial. This is analogous to describing planetary motions as Ptolemaic epicycles relative to a stationary Earth. While this may sometimes be useful (for example, in the gearing of planetarium projectors), it is not conducive to understanding the operative physical laws. Who can say what imaginary effects an imaginary force might have on human physiology? The insights of Kepler and Newton depended on the Copernican conception of Earth as an accelerated reference frame, and viewing it from an inertial reference beyond. The same is true for understanding artificial gravity in rotating structures.

To avoid subscripts, primes, hats, and other difficult-to-read diacritical marks (especially at small font sizes), this derivation has reserved lowercase symbols $(x, y, z, \mathbf{i}, \mathbf{j}, \mathbf{k}, \mathbf{r}, \mathbf{v}, \mathbf{a}, \boldsymbol{\omega})$ for measurements relative to the rotating coordinate system, and uppercase symbols $(X, Y, Z, \mathbf{I}, \mathbf{J}, \mathbf{K}, \mathbf{R}, \mathbf{V}, \mathbf{A}, \boldsymbol{\Omega})$ for measurements relative to the inertial coordinate system. The broader literature on physics and dynamics often reserves some of these symbols for other concepts not relevant to this discussion.

Acknowledgments

I wish to thank my friends, colleagues, and associates, who have provided stimulating discussions over the years that have prompted me to organize a comprehensive response to their misgivings regarding artificial gravity. Though I've been motivated to write this paper by rather strong disagreement with some of them, I have no desire to dishonor anyone. Their comments came in the context of e-mail discussions not intended for publication, so it seems best to omit attributions.

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